

# Search with Learning in the Retail Gasoline Market\*

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## Abstract

This article estimates a model of optimal search where consumers learn the distribution of gasoline prices during their driving trips. Our estimation incorporates traffic information and leverages the ordered search environment to recover parameters of the search and learning process using only station-level price and market share data. We find learning to be a crucial component of search in this market. Consumers' prior beliefs regularly deviate from the true price distribution but are updated quickly following each new price observation. Counterfactuals reveal that these learning dynamics can help explain commonly observed patterns of asymmetric cost pass-through.

*Keywords:* consumer search, consumer learning, dynamic discrete choice model, asymmetric price adjustment, gasoline

*JEL Classification:* D83, L10, L13, L9

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# 1 Introduction

Since Stigler (1961) and McCall (1970), consumer search models have played an important role in explaining imperfectly competitive behavior in many markets. Consumers in these models tradeoff the cost of searching to acquire additional price information against the expected benefit of search, derived from consumers' beliefs about the price distribution. Standard search models rely on the convenient, yet strong, assumption that consumers *know* the true price distribution, thereby simplifying the calculation of consumers' gains from search. In many cases, however, when the prices in a market are unfamiliar to consumers or the price distribution changes regularly with market conditions, consumers are unlikely to know the price distribution with any certainty.<sup>1</sup> Rothschild (1974), Dana (1994), and Benabou and Gertner (1993), among others, have relaxed this assumption, developing theoretical models of search with learning where consumers engage in costly search not only to reveal the prices of particular sellers but also to learn about the actual price distribution. Nevertheless, studies empirically modeling search behavior have largely continued to leverage the assumption that consumers search from a known price distribution. If significant learning is occurring, the search cost distributions estimated in previous empirical studies may be biased, and the broader conclusions drawn about search behavior and market outcomes may be incomplete.

In this article, we relax the assumption of a known price distribution and estimate a model of optimal search by consumers who may be unaware of the true price distribution but update their prior beliefs as they search. We are able to estimate the parameters governing the consumer learning process by taking advantage of the fact that price draws do not occur randomly. In many contexts prices are revealed based on the order in which consumers encounter different sellers as they navigate through the marketplace. For example, consumers observe prices in a specific order as they pass sellers within a market or as they scroll down a list of products on an online shopping website. In our setting, we leverage a crucial observation in the retail gasoline market: consumers are likely to

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<sup>1</sup>For example, using survey data, Matsumoto and Spence (2016) find that students with no prior experience in online textbook purchase tend to expect prices to be higher than observed empirically. Upward-biased price beliefs generally result in less search. On the other hand, students with more experience have more accurate beliefs, consistent with learning.

search and learn the distribution of prices during their driving trips. This feature allows us to recover the parameters of our model from observed prices, station market shares, and the volume of traffic that passes each gas station. Estimation of this model offers several important new insights to the literature, emphasizing the crucial role of learning within the search process. First, in contrast to the known price distribution assumption, we find that consumers' initial priors often differ from the true price distribution, resulting in what we refer to as a *biased prior*. Second, we find that consumers are relatively uncertain about their prior beliefs and, therefore, learn rather quickly from observed prices. Third, the model reveals how biased beliefs and learning influence search behavior and demand. For example, the insights clarify a mechanism through which fluctuations in prices can asymmetrically influence search, helping to explain why cost increases and decreases are passed through asymmetrically in a wide variety of product markets (Peltzman, 2000).

The retail gasoline market is an ideal environment to study consumer search and learning behavior. Frequent price changes resulting from a volatile wholesale cost, as presented in Figure 1, make it difficult for consumers to maintain accurate information on each station's price as well as the distribution of these prices in the market. Our analysis of search introduces two important components that are likely to characterize consumers in this environment. First, consumers are assumed to be uncertain about the price distribution. Second, consumers' prior beliefs are likely to be different from the empirical price distribution. Both components have important implications for search behavior.

Suppose a higher than expected price is observed at a station. A consumer who is uncertain about the price distribution will need to decide whether the high price is specific to this station or reflects a higher than expected price level across all stations. In the latter case, beliefs about the price distribution will be updated based on this high price observation, reducing the expected benefit of additional search. On the other hand, a consumer who knows the price distribution will be more likely to search after observing an unexpectedly high price, recognizing the high price as a station-specific deviation.

Additionally, when consumers do not know the current price distribution, they are likely to form their prior beliefs based on the prices observed during previous purchases or past driving trips. When previous prices differ from the current prices, the resulting bias

in prior beliefs provides a potential explanation for the commonly observed *rockets and feathers* pattern in which cost increases are often passed through to prices more quickly than cost decreases.<sup>2</sup> When prices and costs are rising, consumers' prior beliefs are lower than current prices, increasing consumers' perceived gains from search and causing consumers to search more intensively. As a result, profit margins decrease, and cost increases are passed through more directly to prices. On the other hand, when prices and costs are falling, upward-biased prior beliefs result in underestimated gains of search and less searching, contributing to higher margins and a slower pass-through of cost reductions.

To capture these features, we propose a sequential search model with learning that builds on Rothschild (1974), emphasizing spatial and ex-ante vertical differentiation of sellers. Forward-looking consumers start from diffused prior beliefs likely influenced by prices observed during past driving trips. As consumers encounter a new price observation along their predetermined travel route, they update their beliefs about the price distribution in a Bayesian fashion before deciding whether to purchase gasoline or continue searching. Consumers optimally stop at a station when the realized utility is higher than the continuation value of search conditional on their posterior beliefs. The continuation value of search summarizes the expected value of purchasing at the remaining stations along a route and the alternative of waiting to purchase during a future trip where they might encounter better offers. However, postponing a purchase becomes difficult if one is low on gas. Thus, the search friction in this market takes the form of postponement costs.

To estimate the model, we utilize a novel panel dataset of station-specific prices and quantities for gasoline stations in a small city from December 20, 2014, to May 31, 2016, and combine it with data on traffic flows in the city. Based on the assumption that price search is ordered and directed by driving patterns, we can construct an empirical distribution of search sequences of gasoline stations using the traffic data. Total daily gasoline sales at each station are then modeled as an aggregation of the purchase decisions of individuals searching and learning along different travel routes. Our novel utilization of traffic data also allows us to more realistically model the search behaviors in the retail gasoline market

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<sup>2</sup>Asymmetric pass-through occurs in a broad set of markets (Peltzman, 2000) but is particularly common and well documented in gasoline markets (Borenstein et al., 1997; Lewis, 2011). In addition, Lewis (2011); Yang and Ye (2008); Tappata (2009) provide theoretical models illustrating how imperfect knowledge of the price distribution can cause asymmetric pass-through.

without losing tractability. By replacing the common random sampling assumption with ordered search directed by observed traffic flows, our model allows substitution patterns to depend on the amount of traffic stations share. In addition, we are able to introduce ex-ante vertical differentiation of sellers, allowing consumers to be familiar with the time-invariant characteristics of the stations they regularly encounter along their driving routes.

Our search with learning model nests the standard search model with a known price distribution, providing us the opportunity to statistically test the assumption of a known price distribution in the context of the retail gasoline market. Our estimation results suggest that consumers' prior beliefs are significantly biased by prices from the recent past. Specifically, the average absolute difference between the estimated prior mean and the actual price level is 3.0 cpg, approximately 3.8 times the size of the average day-to-day price change. However, consumers put relatively little weight on these priors, updating beliefs rather quickly and considerably reducing the bias after a few current price observations. The findings reject a known price distribution assumption that assumes both a correct price belief and no learning. They also highlight the importance of accounting for learning when analyzing search behavior. Learning occurs rapidly in our context, and models with learning produce much more accurate predictions of consumer behavior. Estimates from a restricted version of the model with no learning fail to identify prior bias and overestimate the median postponement costs by approximately 34 percent.

Estimating a structural model of search with learning also provides a powerful framework for examining the nature of spatial competition in the market. Demand is estimated to be highly elastic at the station level. A typical station has an estimated own-price elasticity of -8, similar to the findings in Wang (2009). In addition, the traffic data allow the model to generate realistic substitution patterns across stations. If an additional 15 percent of a station's passing traffic has previously driven past a neighbor station, the cross-price elasticity between the two stations increases by 0.65, sufficient to move the station pair from the 5th percentile to the 95th percentile of the cross-price elasticity distribution.

More importantly, station-level demand becomes more elastic when prices are rising than when prices are falling, creating an environment in which cost changes can be passed through asymmetrically. In a counterfactual exercise, we investigate how the learning pro-

cess influences demand asymmetry. We find that when priors are more heavily influenced by past price levels, demand elasticities respond more asymmetrically to price changes. However, conditional on the degree of prior bias, higher prior uncertainty and faster learning leads to less asymmetric demand.

The rest of the article is organized as follows. The next section discusses the related literature. Section 3 introduces the data. Section 4 presents descriptive statistics and key features of the market that motivate our model. The model of search with learning is introduced in Section 5 and estimation strategy and identification are discussed in Section 6. Section 7 presents the estimation results. Section 8 discusses our counterfactual analysis. The last section concludes.

## 2 Related Literature

Our article provides important contributions to the empirical literature on consumer search.<sup>3</sup> Much of this previous work quantifies search frictions and emphasizes their importance in various markets based on the assumption that consumers know the distribution of offers or match values (e.g., Hortacısu and Syverson, 2004; Hong and Shum, 2006; De los Santos et al., 2012; Koulayev, 2014; Honka, 2014; Moraga-González et al., 2018; Nishida and Remer, 2018; Lin and Wildenbeest, 2020). We build on this literature by incorporating consumers who learn about the true price distribution as they search, and show this to be an important aspect of behavior in the retail gasoline market. Our work adds to a new and developing body of empirical research on consumer search with learning. Both Koulayev (2013) and De los Santos et al. (2017) empirically analyze models of search with learning and show that ignoring learning can bias search cost and elasticity estimates. However, they do not estimate the learning process and take prior beliefs as given. In contrast, we develop an empirical strategy to identify both prior uncertainty and prior bias using only aggregate data. Several more recent studies have modeled learning behavior in settings different from ours. Ursu et al. (2020) estimate a sequential search model where consumers search to learn their individual match values for restaurants on a review website. Their estimates suggest a high prior uncertainty that rationalizes the considerable time consumers

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<sup>3</sup>See Ellison (2016) and Honka et al. (2018) for a review of the studies on consumer search.

spent searching each restaurant. Hu et al. (2019) develop a dynamic model of search and Dirichlet learning to study consumers' purchase behavior on Groupon. They find that new consumers have an overly optimistic prior about the distribution of deal quality. Through their interaction with the website over time, consumers have more certain and accurate beliefs about the quality distribution. Consumer learning explains the observed declines in click-throughs and increases in conditional purchase probability. However, unlike our study, estimation in each of these articles requires data on individual's search and purchase histories.

The underlying consumer search process and identification method employed in our model also differ from the existing literature. A number of studies have developed methodologies to estimate search costs using only aggregate data (e.g., Hortaçsu and Syverson 2004; Hong and Shum 2006; Moraga-González and Wildenbeest 2008; Wildenbeest 2011). These studies overcome the curse of dimensionality when integrating over the unobserved search sequences by applying the assumption of random sampling and ex-ante product homogeneity. This article proposes a new estimation method for settings where the order of price observations is directed by how consumers navigate through the marketplace. In particular, we replace the random sampling assumption with variation in search sequences identified using data on driving patterns.<sup>4</sup> With this approach, we can introduce learning and ex-ante seller differentiation into a sequential search setting without losing tractability. Additionally, we can incorporate more realistic substitution patterns between stations. Our search technology also differs from the literature on sequential search for ex-ante differentiated products (e.g., Weitzman 1979; Kim et al. 2010, 2017) in that we model consumer search order as exogenously given by the traffic data rather than endogenously determined by the decreasing order of reservation utilities.<sup>5</sup>

Lastly, our article provides valuable insights into an extensive literature on retail gasoline price dynamics. A large body of empirical research provides reduced-form evidence of asymmetric cost pass-through in the retail gasoline market (e.g., Borenstein et al. 1997;

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<sup>4</sup>Houde (2012) also demonstrates the importance of driving patterns in modeling gasoline demand. Unlike our article, he assumes that consumers have perfect price information and incorporate traffic data into a discrete-choice model of demand.

<sup>5</sup>Our ordered search can be interpreted as a special case of Weitzman's sequential search where the costs of deviating from the current travel route are much larger than the potential gains.

Lewis and Noel 2011; Byrne 2019). Tappata (2009), Yang and Ye (2008), and Lewis (2011) develop theoretical models that rationalize such pricing behavior. Our model closely relates to Lewis’s (2011) reference price search model, where consumers form their price expectations based on past price observations. We allow past prices to influence consumers’ priors within our model of search with learning to reveal how the rockets and feathers phenomenon relates to learning primitives, prior bias and prior uncertainty. Therefore, this article brings together two streams of literature, structural analysis of consumer search and research on cost pass-through in the retail gasoline market.

### 3 Data

In the absence of microdata on individual consumers’ search histories and purchase decisions, we use aggregate data to make inferences about consumers’ search and learning behavior that leads to gasoline purchase. Our sample consists of 46 gasoline stations in a small city, with an urbanized area population of approximately 75,000.<sup>6</sup> The sample period runs from December 20, 2014, to May 31, 2016, for a total of 529 days, during which time we observe the daily price of gasoline at all 46 stations and the daily gasoline transaction volume for 33 of these stations.<sup>7</sup>

We complement the gasoline price and quantity data with data on vehicle traffic flows for our sample region. We use the traffic data to construct an empirical distribution of search sequences of stations in the city. In the following subsections, we describe the three main sources of data used for our empirical analysis.

#### Gasoline Price Data

The per-gallon price of regular unleaded gasoline is collected from two separate gasoline price reporting websites. The primary source is MapQuest.com, an online web mapping ser-

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<sup>6</sup>The city name is not disclosed to protect the identities of the gas stations.

<sup>7</sup>Because our primary focus is to study search behaviors, we exclude 14 stations that do not meet the criteria of a station that competes for gasoline sales among searching consumers. These criteria include a large price board, decent-sized forecourt, and accessible entrance and exit. Almost all excluded gasoline stations are mom-and-pop establishments that operate primarily as convenience stores and have gasoline sales volume lower than the smallest station in the sample.

vice whose gasoline price data are provided by the Oil Price Information Service (OPIS).<sup>8</sup> We record prices from MapQuest.com once per day for every station in the city. Unfortunately, MapQuest (OPIS) does not update every station’s price every day. On average, a station’s price is updated on 54 percent of the sampled days.<sup>9</sup> To address the issue, we complement MapQuest.com’s data with price data collected daily from GasBuddy.com.<sup>10</sup> Unlike MapQuest.com, prices on GasBuddy.com are reported by volunteer spotters in the area. To minimize any issues caused by the potential inaccuracy of prices reported on GasBuddy.com, we only use prices from GasBuddy.com when MapQuest.com does not report the corresponding price for that station on that day.<sup>11</sup> Stations are matched across the two data sources based on the geographic coordinates of the stations, cross-validated with Google Map’s geographic coordinates to ensure accuracy.<sup>12</sup> After merging the price data from these two sources, station prices are missing for only 9.2 percent of the sample days. The remaining missing prices are replaced with the most recent price observed at that station. The average duration over which prices are imputed is 1.6 days.<sup>13</sup> Besides price data, we also obtain information on station characteristics from these sources, including name, brand, address, and geographic coordinates. Moreover, we visit Google Street View and manually collect additional information such as the number of islands and pumps and street conditions for each station.

## Gasoline Transaction Data

Daily station-level expenditure data have been obtained from a major financial services provider for 33 of the 46 stations in our price sample.<sup>14</sup> These data reflect the total dollar

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<sup>8</sup>OPIS obtains price information from credit card transactions and direct feeds from gas stations.

<sup>9</sup>The price coverage rate is slightly lower than other studies that use OPIS data. A possible reason is that the sample city is mid-sized and has more low-volume stations than the major cities studied by other articles. Fewer credit card transactions result in fewer price feeds to OPIS.

<sup>10</sup>Gasoline price data collected from MapQuest.com and GasBuddy.com are widely used in the literature on retail gasoline prices, for example, Lewis and Marvel (2011) and Remer (2015).

<sup>11</sup>Atkinson (2008) shows that prices on GasBuddy.com can identify the features of retail gasoline price competition accurately despite occasional errors.

<sup>12</sup>A station’s name or address cannot be used as a unique identifier for the matching because a station’s name is not unique to a station, and different websites may use different aliases for a street or highway.

<sup>13</sup>Our estimation cannot accommodate missing prices because a station’s price affects many stations’ sales through the traffic network. One missing price will result in a large number of lost observations.

<sup>14</sup>The name of the provider as well as the station names and locations in the data are withheld to protect confidentiality.

amount of purchases made using debit and credit cards associated with the provider’s purchase processing network at each station on each day. Pay-at-pump and in-store purchase totals are reported separately. To eliminate the measurement error caused by non-gasoline transactions, we use pay-at-pump transactions only. A daily measure of the quantity of gasoline purchased at each station is constructed by dividing the total pay-at-pump expenditures by the price of regular unleaded gasoline at the station on that day.<sup>15</sup> Although this quantity measure excludes gasoline purchased with cash or in the store, around 72% of consumers purchase gasoline at the pump (NACS 2016 Retail Fuels Report).<sup>16</sup> Therefore, we believe that our measure of the quantity of gasoline transacted at each station reasonably describes the behavior of consumers searching for and purchasing gasoline.

## Empirical Distribution of Search Routes

As individuals drive along their travel routes, the decision to purchase gasoline at a particular station is affected by the prices observed up to this station as well as the characteristics of the remaining stations along the route. Consequently, search and purchase decisions are modeled at the search-route level, where a search route is defined as a unique ordered sequence of stations visited.

Search routes are assumed to be exogenously determined by consumers’ travel needs. Our analysis uses vehicle travel data to construct an empirical distribution of search routes describing the predicted number of consumers that drive past a specific ordered sequence of stations on an average day. This empirical distribution of travel routes includes two elements: (i) the number of drivers traveling from an origin to a destination and (ii) the route drivers take along the street network connecting the two points. For the first element, we use the origin-destination travel demand estimates for local residents produced by the state Department of Transportation,<sup>17</sup> which report the estimated number of drivers traveling from one Traffic Analysis Zone (TAZ) to another TAZ. The origin-destination

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<sup>15</sup>This construction introduces potential measurement error, as it overestimates the quantity transacted for mid-grade and premium gasoline, which have higher prices. However, it has been estimated that only 15 percent of gasoline transactions are mid-grade or premium.

<sup>16</sup>See Levin et al. (2017) for a detailed discussion of the potential for bias when studying gasoline demand using only payment card purchases.

<sup>17</sup>The Origin and Destination Table is an output of the travel demand model constructed by the Department of Transportation to forecast the traffic in the year 2020.

table spans a seven-county area around the focus city and contains approximately 1800 TAZs. Most TAZs are rather small, with 75 percent of the traffic zones occupying an area of less than  $1.5 \text{ km}^2$ . The larger traffic zones have few residents and are at the fringe of the counties.

To compute the route that drivers take traveling from an origin TAZ to a destination TAZ, we assume that all drivers take the travel route that minimizes driving time. We select the centroid of the origin and destination TAZ as the drivers' start and end locations and determine the single fastest travel route for every origin and destination TAZ pair based on the street network in the area. The ArcGIS Network Analyst package is used to calculate the fastest travel route, with road network data obtained from ArcGIS StreetMap North America.<sup>1819</sup>

Next, we identify the stations along each travel route and the order in which they are passed. Figure 2 provides an example of a travel route connecting a starting location A and an ending location B, including the three stations available to drivers on this route. In many cases, it is difficult for drivers to visit stations on the opposite side of the street because some left-turns cannot be easily made. To consider the potential cost of making left-turns in the model, we also record the side of the street a station is on along each travel route. In the next section, we discuss in detail the different left-turns based on their difficulties.

A search route in our model is formally defined by a specific sequence of stations. As multiple travel routes (origin and destination pairs) may pass the same set of stations in the same order, travel routes are aggregated to the search-route level. An empirical distribution of search routes is constructed by summing up the predicted number of drivers driving past the same ordered set of stations (and only those stations).<sup>20</sup> A total of 991 search routes are identified at the beginning of the sample, and the number increases to 1046 after two additional stations enter the market.<sup>21</sup>

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<sup>18</sup>ArcGIS Network Analyst extension: <https://www.esri.com/en-us/arcgis/products/arcgis-network-analyst/overview>. StreetMap North America: <https://www.arcgis.com/home/group.html?id=ddd06a0bde9c45a1b3e786a2b4e695e8#overview>.

<sup>19</sup>To reduce the computation burden, we grouped the TAZs in each of the surrounding counties into 8 clusters of TAZs based on their locations using the K-Mean algorithm.

<sup>20</sup>Approximately 20 percent of the travel routes that have no stations are excluded. Additionally, we exclude search routes with fewer than 20 drivers traveling per day.

<sup>21</sup>In the structural estimation, the sample periods are divided into three parts based on the entry date.

Although our daily station-level gasoline expenditure data provide many advantages, there are a few limitations. First, expenditure data are only observed for around 70% of the stations in the market. Our model predicts demand at every station, but the identification is based on the stations with observed quantity data. Variation in observed station characteristics is also limited, so the vertical differentiation of sellers is incorporated into the model based on station type. To maintain group size and protect each retailer’s identity, we apply three brand dummies to account for brand heterogeneity, namely, one dummy for major-branded stations and two dummies for retailer brands. The remaining stations are collectively classified as generic stations. Additionally, we include a small station dummy and a large-format station dummy to control for the station scale.

Stations located near Interstate Highway exits also present a challenge. The origin-destination traffic data only describe the travel patterns of local drivers, so potential demand from Interstate drivers is not accurately reflected. Interstate drivers also observe prices and make purchase decisions very differently than the local drivers modeled in this article. To more accurately capture demand at these stations, our model includes a separate dummy variable for each station located at an Interstate Highway exit.

## 4 Retail Gasoline Market Overview

Before introducing the structural model, it is valuable to discuss the features in the retail gasoline market that motivate our modeling choice. More specifically, we first examine the relationship between the station average transaction volume and station characteristics. We then discuss why consumers are likely to have imperfect price information and why it is important to incorporate learning when modeling consumer search in this market.

### Station Transaction Volume and Station Characteristics

Table 1 summarizes the station-level average prices and quantities observed during our sample, as well as some important station characteristics. The top panel provides statistics for all of the stations in the city, whereas lower panels separately consider specific station

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The empirical analysis is based on the empirical distribution of search routes in each period, respectively.

types. Average gasoline prices vary somewhat across stations, exhibiting an interquartile range of 10 cents per gallon around a city-wide average of \$1.61 per gallon (before taxes). Considerably more heterogeneity is exhibited in station-level average transaction volume. Among the 33 stations for which we observe quantity data, the 75th percentile station sells 6.5 times more gasoline than the 25th percentile station. Major-branded stations such as Shell, BP, and Exxon, among others, account for approximately 37 percent of the stations in the city.

On some streets, it may be difficult for drivers traveling in a certain direction to visit stations on the opposite side of the street. For this reason, we classify three types of stations based on left-turn difficulty. Approximately 26 percent of our sample stations can be easily visited by drivers traveling on both sides of the street. These include stations located on two-lane roads or multi-lane roads with a left-turn zone in the center. Another 28 percent of the stations are located where no left-turns are possible because the street has a physical curb or median in the center. The remaining stations are located at major intersections with a traffic light. Casual observation suggests that drivers are likely to forgo possible price savings at these stations to avoid waiting for the left-turn traffic light, especially when the intersection is busy. To provide a conservative measure of the number of consumers each station faces, we define a station's *direct traffic* as the number of drivers who can easily visit the given station, which includes drivers driving on the same side of the street as the station or on the opposite side of the street where a left-turn can be easily made without involving a traffic light. As shown by the last row of the top panel, approximately 11.5 thousand drivers directly drive past an average station on a single day.

Panel (a) of Figure 3 depicts a positive relationship between the direct traffic volume and the transaction volume at a station, both measured in logarithms,<sup>22</sup> revealing that stations passed by more drivers also sell more gasoline. There is significant variation around this relationship, suggesting that other station attributes such as price reputation, brand quality, and station scale also influence station sales. Nevertheless, the pattern demonstrates the advantage of using traffic data to simulate consumers' search patterns for gas stations. Other empirical studies of consumer search (e.g., Hong and Shum 2006; Wildenbeest 2011;

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<sup>22</sup>We exclude the stations at the exit of the interstate highway from this figure because we do not have data measuring highway traffic volume.

Nishida and Remer 2018) have typically adopted an equal-probability random sampling assumption when individual search histories and quantity data are not observed. Our data suggest that consumer search along travel routes better represents consumer behavior than the random sampling assumption.

In recent years, stations with a large number of islands and a large convenience store attached are becoming increasingly popular (e.g., Noel 2016). We group stations into three categories based on their scale, namely, large-format retailers, small-sized stations, and mid-sized stations. More specifically, we define large-format stations as retail stations with at least six islands.<sup>23</sup> All large-format stations in our sample have a sizable convenient store attached. In contrast, small stations have no more than three islands, with a small booth in the center. The remaining stations are categorized as mid-sized stations. The bottom two panels in Table 1 describe the price and quantity distributions for the small and large-format stations. Whereas the average price at large-format stations is, on average, 6 cpg cheaper than at small-sized stations, the average daily sales volume at large stations is 7.8 times greater than at small stations. Notably, large-format stations all have average prices in the lowest quartile of the city distribution, whereas their average sales volumes are all in the highest quartile. The negative correlation between stations' average price and average sales volume is consistent with consumers preferring stations with a reputation of having lower prices.

Our traffic data also reveal that drivers pass enough stations to allow them to search without deviating from their travel routes, as we assume later in the structural model. All stations in our sample display their prices on large signs, so passing traffic in both directions can easily observe prices. Panel (b) and (c) of Figure 3 show the distributions of the number of prices drivers see as well as the number of stations they directly drive past along their travel routes. On average, a driver sees 3.5 prices and directly drives past 2.2 stations along their travel route. Thus, the number of options for drivers is comparable to the number of stores searched by consumers before buying an MP3 player or a book from online retailers.<sup>24</sup>

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<sup>23</sup>An island is an elevated platform where pumps are located. The number of islands provides a better measure on the station scale than the number of pumps. A small station can have four or more pumps cramped on an island, whereas a large-format station generally has two pumps sitting on an island.

<sup>24</sup>Using data on individual online browsing and purchase histories, De los Santos et al. (2017) find

## Two Types of Price Uncertainty

Frequent price changes in the retail gasoline market make it difficult for consumers to maintain accurate price information. Figure 4 shows the distribution of the proportion of stations changing their price from the previous day. On an average day, about 32 percent of stations in the sample change their price.<sup>25</sup> Frequent price changes generate two types of uncertainty in the market: (i) ex-ante uncertainty about the price at each station and (ii) uncertainty about the overall price level in the market. To further analyze the different sources contributing to these two types of uncertainty, we perform the following basic fixed effect regression,

$$p_{jt} = \sum_{j=2}^J \psi_j \text{Station}_j + \sum_{t=1}^T \gamma_t \text{Day}_t + \nu_{jt}, \quad (1)$$

where the price at station  $j$  on day  $t$  is decomposed into a station fixed effect  $\psi_j$ , a day-of-sample fixed effect  $\gamma_t$ , and an idiosyncratic error term  $\nu_{jt}$ .

Using this decomposition, overall variation in price can be viewed as the sum of persistent price differences across stations, captured by station fixed effects  $\psi_j$  and a time-variant component  $\tilde{p}_{jt} = \gamma_t + \nu_{jt}$ . The time-variant price,  $\tilde{p}_{jt}$ , combines the day-of-sample fixed effect,  $\gamma_t$ , which is driven by changes in aggregate market conditions (for example, wholesale cost) common to all stations, and the price residual,  $\nu_{jt}$ , which represents a shock specific to a certain station on a certain day (for example, a private cost shock).<sup>26</sup>

As drivers regularly observe prices during everyday driving, they are likely to be relatively knowledgeable about which stations tend to have higher or lower prices. Consequently, average price differences across stations, captured by station fixed effects  $\psi_j$ , are assumed to be known by consumers in our empirical model. Consumers' uncertainty over prices, therefore, results entirely from variation in prices over time.

The price residuals,  $\nu_{jt}$ , reflect fluctuations in station prices relative to one another (e.g. Lewis 2008). Chandra and Tappata (2011) provide direct empirical evidence of

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consumers visit on average 2.82 online retailers before buying an MP3 player, and De los Santos (2018) finds consumers searched 1.3 online bookstores before purchasing a book.

<sup>25</sup>This number is only a conservative measure of the proportion of stations with price changes on a day due to missing prices. Some price changes are likely not recorded.

<sup>26</sup>Some empirical studies of consumer search consider the price residual as a result of firms employing mixed strategies. This article assumes a pure strategy equilibrium similar to Hortaçsu and Syverson (2004).

such variation, showing that gasoline station pairs exhibit reversals from their normal price ordering approximately 15 percent of the time. This variation creates uncertainty by preventing consumers from knowing a station’s location in the current price distribution prior to search. Empirical studies that structurally estimate models of consumer search focus primarily on this type of uncertainty. The known price distribution assumption assumes that market conditions are constant over time and known to consumers (e.g., Hong and Shum 2006; Wildenbeest 2011; Nishida and Remer 2018).

In reality, however, frequently changing market conditions can make it difficult for consumers to know the level of the overall price distribution, as measured by  $\gamma_t$ . This introduces a second important source of price uncertainty. In this environment, consumers may form expectations of the price level today based on prices observed during past trips or gasoline purchases, resulting in biased beliefs (Lewis 2011). Collecting new price observations allows consumers to learn more about the current distribution, but the presence of station-specific price variation,  $\nu_{jt}$ , prevents them from fully resolving uncertainty in  $\gamma_t$ . Therefore, biased prior beliefs will continue to impact consumers’ posterior beliefs about the current price level, though the weight placed on these priors decreases as more new information is obtained. Incomplete knowledge of the price distribution can have important impacts on consumer behavior. For example, biased beliefs provide one explanation for why wholesale gasoline cost increases are often passed through to retail prices more quickly than cost decreases, as depicted in Figure 1.

Based on the decomposition in Equation 1, Table 2 presents the relative magnitudes of the two components of price variation that give rise to uncertainty. The price residuals,  $\nu_{jt}$ , have a standard deviation of 3.2 cpg, confirming the presence of substantial within-day price dispersion. In addition, the price levels,  $\gamma_t$ , exhibit considerable fluctuation during our sample period, spanning from a minimum of \$1.08 to a maximum of \$2.04 before taxes. From one day to the next, the average absolute difference in the price level is 0.8 cpg. However, the discrepancies between the priors and the actual price levels might be much larger, as consumers are likely to use the price at their last gasoline purchase as a reference price (Lewis, 2011). For example, the average absolute difference between the current price and the price seven days prior is 4.5 cpg.

We now provide some descriptive evidence on whether consumers know about the current price distribution under the premise of consumer search. If consumers have correct knowledge about the actual price distribution, past prices should not affect consumers' search decisions. In contrast, when consumers are uncertain about the actual price distribution and formulate their price expectations based on prices acquired from past driving trips or purchases, these past prices may bias consumers' perceived benefit of price search and influence consumer search in certain directions. In particular, lower past prices may bias consumers' price expectations downward, causing more consumers to postpone their purchases to future trips searching for better prices, consequently lowering current gasoline sales. Similarly, higher past prices may reduce the perceived benefit of search or postponement, leading to more purchases on current driving trips. To investigate the relationship between purchases and past prices, we regress the logarithm of a station's daily transaction volume on its own price, its closest competitors' prices, and the average price level in the city seven days prior while controlling for station as well as day of week and month of sample fixed effects.<sup>27</sup> We measure a station's closest competitors in terms of the amount of traffic the stations have in common. Note that under imperfect price information, price changes at subsequent stations along a travel route are unknown to consumers and thus do not affect their search or purchase decisions at the current station. In other words, a price change at an upstream station can influence the demand at a downstream station but not the other way around. Therefore, we define a station's common traffic shared with a neighbor station as the proportion of the given station's passing traffic (in all directions) that has previously passed the neighbor. We then rank neighbors in terms of common traffic shares for each station.<sup>28</sup> Table 3 provides the coefficient estimates for the panel regression. A typical station's daily gasoline sales decrease in its own price and increase in the prices of its closest two neighbors as expected. Importantly, the coefficient on the logarithm of the past price level is positive and significant at the 1% level in all specifications. The data pattern is consistent with consumers' behaviors when they are uncertain about the current

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<sup>27</sup>We use the price level seven days ago,  $\gamma_{t-7}$ , as a measure of the prices that consumers observe on their past trips or purchases. We have estimated the model using the price level on various days prior and obtained similar results.

<sup>28</sup>The magnitude of these common traffic shares is discussed later when we consider station pair characteristics in Section 7.

price level. However, it contradicts the assumption of a known distribution, where past prices should not affect consumers' search and purchase decisions.

## 5 Model

Based on the important institutional details of the retail gasoline market, we specify a model of price search in which consumers are uncertain about the current price distribution and learn about the distribution as they observe prices. The model considers heterogeneous and forward-looking individuals, each traveling along a particular (exogenously determined) route, sequentially encountering a known set of stations, perhaps from home to work or from home to a store. Consumers hold prior beliefs about the average price level in the market, likely based on the prices observed from past driving trips or purchases. As a consumer passes each station, she updates her price beliefs before optimally deciding whether to stop and purchase or continue on to potentially purchase at a subsequent station.<sup>29</sup> Consumers also have the option of postponing purchase until a future trip but incur a postponement cost that varies across consumers to reflect that some consumers need to purchase gasoline more urgently than others. Therefore, the probability that a consumer will purchase from a station depends on the realized utility of purchasing at the observed price and the expected value of continuing to search given her posterior belief of the price distribution.

Although our search model characterizes an individual consumer's purchase decision, our empirical model will be estimated using station-level quantity and price data. A station's potential customers may be traveling along many different routes and encountering different sets of competing stations. The quantity of gasoline sold at a particular station can then be modeled by aggregating individual predicted purchase decisions over the empirical distribution of consumers across search routes. The following subsections detail the different components of the individual search model. Then, in the next section, we discuss the construction of the empirical model, including aggregation to the station level and the additional assumptions required for estimation.

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<sup>29</sup>Although gasoline consumption does respond somewhat to changes in price, this study focuses instead on how consumers decide where and when to make that gasoline purchase. In our empirical estimation, we use time fixed effects to control for the changes in overall demand level as detailed in Section 6.

## Utility

Demand for gasoline is characterized by consumers searching the prices of stations along their travel routes. We consider a city containing a set,  $\mathbb{J}$ , of  $J$  stations indexed  $j = 1, 2, \dots, J$ . Consumers each demand 10 gallons of gasoline.<sup>30</sup> Following Hortaçsu and Syver-son (2004), we assume consumers have an indirect utility for gasoline at station  $j$  on day  $t$  equal to:

$$u_{jt} = X_j\beta - p_{jt},$$

where  $X_j$  represents station  $j$ 's non-price characteristics, and  $p_{jt}$  is the unit cost of gasoline (per gallon price multiplied by 10 gallons). The coefficient on the price is normalized to  $-1$ , so utilities are expressed in dollar value.

Based on the price decomposition in Equation 1, we can rewrite the indirect utility as

$$\begin{aligned} u_{jt} &= X_j\beta - \psi_j - \gamma_t - \nu_{jt} \\ &= V_j - \tilde{p}_{jt}, \end{aligned} \tag{2}$$

where  $\gamma_t$  represents the daily average price level in the city,  $\psi_j$  captures the persistent price difference between stations, and  $\nu_{jt}$  is the idiosyncratic deviation of station  $j$ 's price on day  $t$  from its own average and the city average. Therefore, utility can be partitioned into two components: the value of station  $j$ 's time-invariant characteristics,  $V_j = X_j\beta - \psi_j$ , and a time-varying price component,  $\tilde{p}_{jt} = \gamma_t + \nu_{jt}$ .

The partition of the utility function is motivated by the features of the retail gasoline market. Repeated observations and frequent purchases at a number of stations allow consumers to become aware of the station characteristics that are constant over time. These include the station's location and brand, as well as its reputation for being a high or low priced station (represented in the model by  $\psi_j$ ). The  $V_j$  component then represents the part of utility known to consumers before search. In contrast, time-variant prices, representing the changes in prices over time and across stations, are unknown to consumers, as discussed in the previous section.

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<sup>30</sup>A unit of gasoline purchase of 10 gallons is a scaler chosen for the convenience of interpreting the estimation results.

## Consumer Learning and Prior Belief

This subsection describes the evolution of consumers' price beliefs as they travel along their driving trips on day  $t$ . Because retail gasoline prices frequently rise and fall in response to volatile wholesale prices, consumers are uncertain about the distribution of the time-variant prices. This article focuses on consumers' uncertainty about the price level, a crucial component of the price distribution. We capture this uncertainty by assuming consumers hold some common prior beliefs about the price level.<sup>31</sup> In particular, we assume a normal distribution for the prior beliefs. Rather than knowing the actual price level,  $\gamma_t$ , consumers perceive possible price levels as random variables,  $m_{0t} \sim N(\mu_{0t}, \sigma_{\mu_{0t}}^2)$ . Here  $\mu_{0t}$  is the mean (expectation) of the perceived price levels, later referred to as the prior mean, and  $\sigma_{\mu_{0t}}^2$  measures the degree of initial uncertainty about the price level.

We further assume the price residuals,  $\nu_{jt}$ , follow a normal distribution. Thus,  $\tilde{p}_{jt} \sim N(\gamma_t, \sigma^2)$ , where  $\sigma^2$  denotes the magnitude of actual price dispersion.<sup>32</sup> Based on past experiences, consumers are likely familiar with the typical level of idiosyncratic price variability in the market. Therefore, we assume that consumers know  $\sigma^2$ .<sup>33</sup> Due to relative price variation, each price observation only provides a noisy signal of the true price level. To better illustrate the relative importance between the prior and the new price observations in formulating the posterior beliefs, prior uncertainty,  $\sigma_{\mu_{0t}}^2$ , can be expressed in terms of  $\sigma^2$ , and thus the prior belief is

$$m_{0t} \sim N\left(\mu_{0t}, \frac{\sigma^2}{\alpha_0}\right). \quad (3)$$

Here  $\alpha_0$  is commonly known as the prior weight, which is inversely related to the prior uncertainty.

As consumers observe new prices, they update their beliefs about the price level. Let  $x_n$  be the realization of the  $n$ th time-variant price observation. According to Bayes' rule, the

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<sup>31</sup>The common prior assumption is for analytical tractability. However, as consumers observe different prices along different search routes, their posterior beliefs become different.

<sup>32</sup>We make two important assumptions here. First, the normality assumption is useful because the conjugate prior of a normal distribution is itself. Second, we assume that the price dispersion is constant over time. Although the degree of gasoline price dispersion has been shown to fluctuate over time when price levels change (Lewis and Marvel, 2011; Chandra and Tappata, 2011), our article abstracts from this second-order effect.

<sup>33</sup>This assumption is also necessary for the identification of the prior weight (Mehta et al., 2003; Ursu et al., 2020). In the analysis,  $\sigma$  is set to 0.32 to match the empirical distribution presented in Table 2.

posterior belief about the price level after observing  $n$  prices follows a normal distribution

$$m_{nt} \sim N\left(\mu_{nt}, \frac{\sigma^2}{\alpha_0 + n}\right), \quad (4)$$

where

$$\mu_{nt} = h(\mu_{n-1,t}, n, x_n) = \frac{(\alpha_0 + n - 1)\mu_{n-1,t} + x_n}{\alpha_0 + n}. \quad (5)$$

The posterior uncertainty,  $\frac{\sigma^2}{\alpha_0 + n}$ , falls in the number of price observations. Based on Equation 5, the posterior mean of the perceived price level can also be expressed as a weighted average of the prior mean and the new price observations,

$$\mu_{nt} = \frac{\alpha_0}{\alpha_0 + n}\mu_{0t} + \frac{1}{\alpha_0 + n} \sum_{k=1}^n x_k. \quad (6)$$

The posterior belief, which captures the learning process, depends on two critical components: prior uncertainty and prior mean. The prior weight,  $\alpha_0$ , determines the speed of learning. When  $\alpha_0$  is smaller, meaning that consumers are more uncertain about their prior beliefs, the posterior mean is updated more by each price observation. On the other hand, a larger  $\alpha_0$  suggests a slower update, as the posterior mean depends more on the prior mean. Moreover, when consumers are perfectly certain of their prior beliefs about the price level ( $\alpha_0$  is infinite), the posterior mean always equals the prior mean regardless of the new price observations,  $\mu_{nt} = \mu_{0t}$ . In other words, no learning occurs, and consumers believe that any observed price deviation from their prior mean is the result of a station's specific price change rather than a market-level price change.

The prior mean,  $\mu_{0t}$ , also plays a vital role in formulating the posterior beliefs. Importantly, the prior mean does not need to equal the actual price level,  $\gamma_t$ . In fact, as discussed in Section 4, prior beliefs are likely biased as consumers formulate their prior beliefs based on the prices observed from previous gasoline purchases or driving trips.

## Ordered Search

Drivers typically pass multiple gas stations while driving to their desired destinations. As a result, unlike some other product markets, consumers can sequentially search the prices

of multiple stations with zero search cost. In practice, drivers rarely alter their routes or make separate trips to visit additional stations. Hence, we adopt a model that assumes such deviations from the travel route, including recall (driving back to a previously passed station), are too costly. Consumers' price search is sequential and ordered, as they know ex-ante the predetermined order in which they will pass a specific set of differentiated stations.

We assume that consumers update their beliefs based on the price observations from both sides of the street. In contrast, visiting a station may be more difficult for consumers on routes that pass on the opposite side of the street from the station. We capture this by introducing a visit cost (or turn cost),  $\tau_{rn} \in \{0, \tau, \infty\}$ . This cost is zero if the traveler is on the same side of the street as the station or if a left-turn is easy to make, but will take on a non-negative value,  $\tau$ , if a left-turn requires waiting for traffic lights at a major intersection. For travelers who are unable to visit the station due to left-turn restrictions this turn cost parameter becomes infinite.<sup>34</sup> With some abuse of notation, let  $r(n)$  return the station index  $j$  for the  $n$ th station along route  $r$ . This station's route-specific ex-ante known utility is then

$$V_{rn} = V_{r(n)} - \tau_{rn}. \tag{7}$$

Consider a consumer  $i$ 's search decision as she drives along a route  $r$  on day  $t$ . For notational simplicity, the day index  $t$  is suppressed until necessary. As the consumer drives to each station, she costlessly observes the price. She updates her belief about the prices at the other stations before deciding whether to purchase gasoline at this station or go to the next one. This decision amounts to an optimal stopping problem involving a value function,  $W_r$ , with three state variables: the number of prices already observed,  $n$ , the price at the current station,  $x_{rn}$ , and the posterior mean,  $\mu_{rn}$ . Upon observing the price at a station  $n$  prior to the final station on the route, the consumer trades off the realized utility at the  $n$ th station with the value of continuing searching, evaluated based on her current estimates of the price distribution given the price information obtained. Therefore,

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<sup>34</sup>See Section 4 for additional discussion of how left-turn difficulty is determined for each station.

the value function can be recursively defined as,

$$W_{irn}(\mu_{rn}, x_{rn}) = \max \left\{ V_{rn} - x_{rn}, \int W_{irn+1}(h(\mu_{rn}, n+1, x_{rn+1}), x_{rn+1}) \cdot dF_{rn}(x_{rn+1}) \right\}, \quad (8)$$

where  $F_{rn}(x_{rn+1})$  is the posterior predictive distribution of possible unobserved prices at the  $n+1$ th station given the  $n$  prices already observed along route  $r$ . As we show in Appendix A, it follows a normal distribution with  $x_{rn+1} \sim N\left(\mu_{rn}, \sigma^2 + \frac{\sigma^2}{\alpha_0+n}\right)$ . The predictive distribution takes into account both the station-specific price variation,  $\sigma^2$ , conditional on a possible price level as well as the posterior uncertainty over the price levels,  $\frac{\sigma^2}{\alpha_0+n}$ .

In practice, drivers typically travel on a variety of different routes to and from their various destinations. Some may choose not to purchase on their current trip, instead continuing to search for a better deal on a future trip along a different route. This option becomes increasingly costly, however, when a consumer is close to running out of gas. In our model, if a consumer does not purchase from a station along the current travel route, she pays a postponement cost  $c_i$ . This  $c_i$  will be higher for those who need to purchase now, and lower for those seeking to buy gasoline but not under pressure to do so immediately.<sup>35</sup> Because our data do not track individuals' driving behaviors over time, we assume that consumers face the same set of routes  $\mathbb{R}$  and probabilities  $\lambda_r$  given by the traffic data when they choose to postpone and start the search over.

Therefore, at the final station  $n = N_r$ , the value function becomes

$$W_{irN_r}(\mu_{rN_r}, x_{rN_r}) = \max \left\{ V_{rN_r} - x_{rN_r}, -c_i + \sum_{r' \in \mathbb{R}} \lambda_{r'} \cdot \int W_{ir'1}(h(\mu_{rN_r}, 1, x_{r'1}), x_{r'1}) \cdot dF_{rN_r}(x_{r'1}) \right\}. \quad (9)$$

The continuation value of search at the end of a search route is then the sum of the postponement cost and the weighted sum of the expected value function at the start of a new travel route. We assume consumers are myopic. When considering postponement, consumers perceive the expectation of the future price level to be the same as the expectation

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<sup>35</sup>The postponement cost can be interpreted as the psychological cost of worrying about running out of gasoline or a reduced-form alternative of the future stock-out cost.

of the price level based on their subjective posterior beliefs at the end of a route.<sup>36</sup> However, consumers' uncertainty about the future price level is reset to  $\sigma^2/\alpha_0$  ( $n = 0$ ), as they have not yet observed any prices on the next travel route. Therefore,  $F_{rN_r}(\cdot)$  is a normal distribution with mean  $\mu_{rN_r}$  and variance  $\sigma^2 + \frac{\sigma^2}{\alpha_0}$ .

Conditional on consumer taste and learning parameters  $\theta$ , we denote the continuation value of search at any station  $n < N_r$  along route  $r$  as

$$Z_{rn}(\mu_{rn}, c_i|\theta) = \int W_{irn+1}(h(\mu_{rn}, n+1, x_{rn+1}), x_{rn+1}) \cdot dF_n(x_{rn+1}) \quad (10)$$

$$= \int \max\{V_{rn+1} - x_{rn+1}, Z_{rn+1}(h(\mu_{rn}, n+1, x_{rn+1}), c_i|\theta)\} \cdot dF_{rn}(x_{rn+1}), \quad (11)$$

where Equation 11 is obtained by combining Equation 8 and Equation 10. At the final station  $n = N_r$ ,

$$\begin{aligned} Z_0(\mu_{rN_r}, c_i|\theta) &= Z_{rN_r}(\mu_{rN_r}, c_i|\theta) = \\ &= -c_i + \sum_{r' \in \mathbb{R}} \lambda_{r'} \int \{V_{r'1} - x_{r'1}, Z_{r'1}(h(\mu_{rN_r}, 1, x_{r'1}), c_i|\theta)\} \cdot dF_{rN_r}(x_{r'1}). \end{aligned} \quad (12)$$

**Proposition 1** *The continuation value of search can be written as  $Z_{rn}(\mu_{rn}, c_i|\theta) = z_{rn}(c_i|\theta) - \mu_{rn}$  for any  $r \in \mathbb{R}$  and  $n = 1, 2, \dots, N_r$ . Solving the recursive relationship presented in Equation 11 and 12, the continuation value of search can be simplified as follows:*

When  $n < N_r$ ,

$$z_{rn}(c_i|\theta) = z_{rn+1}(c_i|\theta) + \sigma \sqrt{\frac{\alpha_0 + n}{\alpha_0 + n + 1}} \cdot (\zeta_{rn+1} \cdot \Phi(\zeta_{rn+1}) + \phi(\zeta_{rn+1})), \quad (13)$$

and when  $n = N_r$ ,

$$z_{rN_r}(c_i|\theta) = z_0(c_i|\theta) = -c_i + \sum_{r' \in \mathbb{R}} \lambda_{r'} \left[ z_{r'1}(c_i|\theta) + \sigma \sqrt{\frac{\alpha_0}{\alpha_0 + 1}} \cdot (\zeta_{r'1} \cdot \Phi(\zeta_{r'1}) + \phi(\zeta_{r'1})) \right], \quad (14)$$

where  $\zeta_{rn+1} = \frac{V_{rn+1} - z_{rn+1}(c_i|\theta)}{\sigma \sqrt{\frac{\alpha_0 + n}{\alpha_0 + n + 1}}}$ .  $\Phi(\cdot)$  and  $\phi(\cdot)$  are the CDF and PDF of the standard normal distribution respectively.

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<sup>36</sup>This is a realistic assumption, as the average price level series follows a random walk.

Proposition 1 shows that the continuation value of search  $Z_{rn}$  is the sum of posterior mean,  $\mu_{rn}$ , and a function of the postponement cost,  $z_{rn}$ , conditional on the consumer parameters (see Appendix B for proof). In other words,  $z_{rn}(c_i|\theta)$  summarizes the value of the time-invariant characteristics of the remaining options along a route for the consumer with postponement cost  $i$ . Based on the recursive relationship, we can numerically solve for  $z_0$ , and subsequently  $z_{rn}$  as a function of  $c_i$  at any station along any route. In practice, the solution is given by linear interpolation.

Having not purchased at any previous stations on the route, purchase occurs at the  $n$ th station if  $V_{rn} - x_{rn} \geq z_{rn}(c_i|\theta) - \mu_{rn}$ , where  $\mu_{rn} = \frac{\alpha_0}{\alpha_0+n}\mu_0 + \frac{1}{\alpha_0+n} \sum_{k=1}^n x_k$ . It is straightforward to show that  $z_{rn}$  is decreasing in  $c_i$  for any  $r \in \mathbb{R}$  and  $n = 1, 2, \dots, N_r$ . However, the function  $z_{rn}(c_i|\theta)$  is bounded from below by the the maximum expected utility of the remaining stations. To see this, we have  $z_{rn} \geq z_{rn+1}$  and  $z_{rn} \geq V_{rn+1}$  for any  $n < N_r$  and  $c_i$  from Equation 13. Therefore, as long as the realized utility net of posterior mean is greater than the lower bound, given by  $z_{rn}(+\infty|\theta)$ , the critical postponement cost  $c_{rn}^*$  that makes the consumer indifferent between purchasing and continuing to searching satisfies the following,

$$\begin{aligned} z_{rn}(c_{rn}^*|\theta) &= V_{rn} - x_{rn} + \mu_{rn} \\ &= V_{rn} - \frac{\alpha_0 + n - 1}{\alpha_0 + n} x_{rn} + \frac{1}{\alpha_0 + n} \sum_{k=1}^{n-1} x_{rk} + \frac{\alpha_0}{\alpha_0 + n} \mu_0. \end{aligned} \quad (15)$$

If the realized net utility is less than the lower bound, the consumer will continue searching. In this case,  $c_{rn}^*$  becomes infinite so that the consumer does not purchase at the current station.

Intuitively, for the consumer to optimally purchase at a station, her postponement cost must be large enough to make the realized utility greater than the continuation value of search. Therefore, the lower bound of postponement cost necessary for the consumer to stop searching is  $c_{rn}^*$ . Additionally, suppose the consumer has already driven past at least one station along the route. For the consumer to optimally purchase at the current station, her postponement cost should not be so large that she has already purchased at a previous station. Therefore, we denote the upper bound of postponement cost as

$c_{rn}^{**} = \min(c_{r1}^*, \dots, c_{rn-1}^*)$  when  $n > 1$ . At the first station, we define  $c_{rn}^{**} = \infty$  when  $n = 1$ , so that the consumer will purchase if  $c_i \geq c_{r1}^*$ .

Conditional on searching along route  $r$ , the proportion of consumers who purchase from the  $n$ th station is

$$q_{rn} = \begin{cases} G(c_{rn}^{**}) - G(c_{rn}^*) & \text{if } c_{rn}^{**} \geq c_n^* \\ 0 & \text{otherwise,} \end{cases} \quad (16)$$

where  $G(\cdot)$  is the CDF of the postponement costs.

Equation 16 shows that the conditional purchase probabilities along a route are given by the postponement cost distribution evaluated at the critical values. Equation 15 establishes the relationship of the critical values with seller utilities and the posterior beliefs resulting from consumer learning. In the next section, we discuss how we apply data to estimate the model parameters.

## 6 Estimation

We combine gasoline data with traffic data for our estimation. Both data sources are aggregated and do not provide information on an individual's driving or purchase history within a day or across days. As a result, two important assumptions are necessary to estimate the model. First, we assume that the gasoline transactions are made by a new group of drivers each day. Therefore, the model is estimated at the day level. Second, all consumers start each day with the same prior belief. Motivated by the discussion in Section 4, the prior mean is parameterized as a weighted average of the price level seven days ago and the current price level:

$$\mu_{0t} = \pi\gamma_{t-7} + (1 - \pi)\gamma_t. \quad (17)$$

The prior bias  $\pi$ , a variable from 0 to 1, captures the influence of price observations from past driving trips or gasoline purchases on prior price expectations.

This specification of the learning process closely relates to the existing search literature. Due to data limitation, most existing empirical studies on search with learning, including Koulayev (2013) and De los Santos et al. (2017), do not estimate the parameters governing

the learning process and assume a correct prior belief ( $\pi = 0$ ) and a prior weight equal to the number of product and seller combinations. Similar to Hu et al. (2019), we estimate the learning process. However, our focus is on how much the past prices bias the prior mean rather than estimating the prior mean itself. Our specifications also allow us to empirically distinguish alternative search model assumptions based on our data. The prior bias  $\pi$  allows us to statistically test whether consumers have correct expectations about the price distribution ( $\pi = 0$ ) or use past price observations as a reference price ( $\pi = 1$ , e.g. Lewis 2011). When  $\pi = 0$  and  $\alpha_0 = \infty$ , our model nests the standard search models with a known price distribution.

We further aggregate the purchase decisions made by drivers searching along their respective search routes to construct each station’s daily market share, matching the observation level of our gasoline data. More specifically, we first define a station’s observed market share as the share of total drivers who purchase gasoline at that station on a day. The number of drivers who purchase at a station is measured by the daily quantity of gasoline transacted at the station divided by the unit amount of gasoline per purchase. The total number of drivers is obtained from the traffic data. Second, we calculate our model predicted market shares. On any given day, only a small share of the drivers purchase gasoline because most already have sufficient gasoline remaining in their tanks. To capture the behaviors of the drivers who do not consider a purchase in our model, we allow the distribution of postponement costs to have a probability mass of  $1 - \eta_t$  at zero.<sup>37</sup> The remaining  $\eta_t$  share of drivers have a positive probability of purchasing gasoline on their search routes. Note that  $\eta_t$  includes a set of day of week and month of sample dummy variables to account for the changes in overall demand (frequency of purchase) for gasoline over time. For example, during summer seasons, as drivers purchase more frequently,  $\eta_t$  also goes up. We assume the positive postponement costs follow a log-normal distribution,

$$\ln c_i = \mu_c + \varepsilon_i, \tag{18}$$

where  $\mu_c$  is a constant and  $\varepsilon_i$  is a stochastic term following a standard normal distribution.

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<sup>37</sup>Because the price distribution follows a normal distribution and is not bounded, the value of continuing to search is infinite when the postponement cost is zero. Consequently, the drivers with zero postponement cost will never make a purchase.

Therefore, following Equation 16, the share of total consumers who travel on route  $r$  and purchase from the  $n$ th station in period  $t$  can be rewritten as

$$q_{rnt} = \begin{cases} \eta_t(\Phi(\ln c_{rnt}^{**} - \mu_c) - \Phi(\ln c_{rnt}^* - \mu_c)) & \text{if } c_{rnt}^{**} > c_{rnt}^* \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

Let  $r^{-1}(j)$  return the location index  $n$  of station  $j$  on route  $r$ . We obtain the predicted market share of station  $j$  at time  $t$  by aggregating the conditional purchase probabilities across all the routes the station is on,

$$\hat{Q}_{jt} = \sum_{r \in \mathbb{R}} \lambda_r q_{r(r^{-1}(j))t} \mathbf{1}(j \in r). \quad (20)$$

Equation 20 shows how we map the observed market share at a station into the conditional purchase probabilities resulting from consumers' search and learning decisions. This relationship allows us to estimate the model using nonlinear least squares. More specifically, to estimate the model, we first use the regression results from Equation 1 to separate prices into station-average prices and time-variant prices. Whereas consumers know ex-ante the station-average price along with other station characteristics, they search to realize the time-variant price at each station and update the posterior mean. Second, we calculate the purchase probability at the station-route level. Given a set of parameters, we can numerically solve for  $z_0()$ , an unknown function of  $c_i$ , over a set of equations described by Equation 14 and 13 using the fixed point algorithm. Then we can obtain a numerical solution for  $z_{rn}()$  as a function of  $c_i$  for each station along each route. All solutions are given by linear interpolation on a grid of  $c_i$ . For each station along a route, we can calculate the realized utility net of the posterior mean and determine the critical  $c_{rn}^*$  that makes an individual indifferent between purchase and searching using interpolation on the grid of  $z_{rn}()$ . Equation 16 specifies the probability that consumers on a route will purchase at a station given these critical postponement cost levels. Finally, we aggregate the conditional purchase probabilities over the empirical distribution of search routes to obtain the model predicted market shares according to Equation 20. We choose the set of parameter values to minimize the sum of squared deviations between the observed and predicted market

shares.

The parameters to be estimated are consumer preferences,  $\beta$ s, the postponement cost,  $\mu$ , the prior belief, including prior bias,  $\pi$ , and prior weight,  $\alpha_0$ , and a set of time fixed effects. To illustrate the identification of these parameters, we start with a simplified model—only one station on each route. Let  $\lambda_j = \sum_{r' \in \mathbb{R}} \lambda'_r \mathbb{1}(j \in r)$  denote the total share of traffic that goes past station  $j$  and  $z_j$  as the function of the continuation value of search at station  $j$  net of the mean perceived price level. Based on Equation 14, the critical postponement cost that solves Equation 15 is

$$\begin{aligned} \zeta_{jt}^* &= z_j^{-1} \left( V_j - \frac{\alpha_0}{\alpha_0 + 1} x_{jt} + \frac{\alpha_0}{\alpha_0 + 1} \mu_{t0} \right) \\ &= \sigma \sqrt{\frac{\alpha_0}{\alpha_0 + 1}} \sum_{j' \in \mathbb{J}} \lambda_{j'} \cdot [\zeta_{j't}^* \cdot \Phi(\zeta_{j't}^*) + \phi(\zeta_{j't}^*)], \end{aligned} \quad (21)$$

where  $\zeta_{j't}^* = \left( \frac{V_{j'} - V_j}{\sigma \sqrt{\frac{\alpha_0}{\alpha_0 + 1}}} + \sqrt{\frac{\alpha_0}{\alpha_0 + 1}} \frac{x_{j't} - \mu_{t0}}{\sigma} \right)$ . Holding prior weight  $\alpha_0$  constant, consumer preferences and postponement costs can be separately identified. Because the sampling probabilities are observed from the traffic data, the relative positions of station average market shares inform us about the degree of seller differentiation. As day to day price fluctuations cause station market shares to deviate from these averages, we are able to pin down the differences in station utilities and thus consumer preference parameters and the postponement cost distribution parameter. In theory, the prior weight can be estimated by the function form in this simplified model, as it enters Equation 21 non-linearly, though the identification may be weak. In the full model, the identification depends on the search sequences provided by the traffic data. Equation 15 imposes a structure on how the price observations along a route and the prior belief affect the conditional purchase probability of a station. As a result, the variation in a station's market share due to the neighboring stations' price changes identifies the prior weight,  $\alpha_0$ . Lastly, past prices influence the purchase probabilities via prior beliefs. Conditional on the prior weight, the prior bias  $\pi$  is identified by how much the market shares are affected by past prices. The Monte Carlo simulation presented in Appendix C also confirms that our estimation approach can separately identify our model parameters.

## 7 Results

To facilitate comparison, we estimate our full search model with learning as well as a restricted version that does not incorporate consumer learning (i.e.,  $\alpha_0 = +\infty$ ). The results are presented in Table 4, with estimates from the full model in Column (1) and estimates from the restricted model in Column (2). Estimates of the bias and learning parameters in Column (1) reveal that consumers' initial beliefs regarding the distribution of prices exhibit significant bias. But as consumers observe new prices, they update their beliefs relatively quickly. More specifically, the bias parameter suggests that, before observing any prices, 67% of a consumer's prior belief depends on prices observed in the prior period. This statistically significant weight on the past prices rejects the common assumption that consumers behave as if they have a correct expectation about the price distribution. The average absolute difference between the estimated prior mean and the actual price level is 3.0 cpg, approximately 3.8 times the size of the average day-to-day price change. However, this large initial bias is quickly moderated as the consumer observes new prices along her travel route. The estimated weight on the initial prior is 0.30, suggesting a fast rate of learning. For example, after one new price observation, the bias reduces by 77% to 0.7 cpg, and after two new price observations, the bias reduces by 87% to 0.4 cpg in expectation. This rapid learning is consistent with the fact that the distribution of gasoline prices changes regularly in response to wholesale cost volatility. As a result, prices observed in the prior period carry only limited information, whereas prices sampled today are much more informative about the current price level.<sup>38</sup>

With regard to station attributes, the estimates suggest that consumers purchasing 10 gallons of gasoline are willing to pay \$1.09 more to avoid waiting for the left-turn signal at a busy intersection. Consumers also appear to value the features offered at large-format stations, placing a \$0.70 premium on purchasing gas at these stations. In contrast, the willingness to pay for a gas purchase at a small station is \$1.17 lower than at a medium-sized station. We also find that consumers are willing to pay \$0.52 more at a major branded station than at a generic station, all else constant.

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<sup>38</sup>The estimated speed of learning is much faster than what is previously assumed by the literature. For example, Koulayev (2013) and De los Santos et al. (2017) choose the prior weight to be the number of product-retailer combinations, resulting in a much slower rate of learning.

Based on the postponement cost estimates, for drivers who have a positive probability of purchasing gasoline, the median cost of postponing a purchase to a future trip is \$0.58, and 25 percent of these drivers are willing to pay \$1.13 more to purchase on the current route rather than to postpone. Note that the postponement cost reflects the trade-off between purchase on the current trip and purchase on a future trip where a consumer is not only uncertain about the prices and their distribution but also the set of stations she will drive past.

In the restricted version of the search model with no learning (Column 2 of Table 4), the prior weight in Equation 3 is set to positive infinity, meaning that a driver's posterior belief about the price level always equals to her prior belief. However, we still allow for the possibility that past prices may bias the initial prior. Without learning, any bias that is present will persist and influence purchase decisions regardless of how many new prices a consumer observes. As one would expect, the search model without learning does not fit the data as well as the full model. In addition, the bias parameter estimate becomes much smaller and statistically indistinguishable from zero. A comparison to the results of the full model is particularly informative here. When learning is incorporated into the search model, estimates reveal that substantial bias in consumers' priors can arise but is quickly mitigated through learning. In other words, bias may influence a consumer's expectations when visiting the initial stations along the travel route, but will have little impact when visiting subsequent stations. The restricted search model without learning assumes that expectations remain fixed throughout, therefore, making it impossible to identify the presence of biased priors for a subset of stations on the travel route.

The estimates of the postponement costs are also very different. The median postponement cost is \$0.77 from the no-learning model, approximately 34% higher than the estimate in the learning model. The higher postponement costs in the no-learning model make sense within our theoretical context (Equation 15). With no learning, consumers behave as if they are certain about the price distribution and how it compares to the current price observation so that this model predicts greater responsiveness to price changes. As a result, the estimated postponement cost parameter will be inflated to allow the no-learning model to fit the relatively low level of price responsiveness observed in the data.

## Own-Price Elasticities

We next investigate consumers' predicted responses to station-specific price changes based on the search with learning model as well as the no-learning model. Price elasticities are obtained by simulating how station-specific gasoline purchases change following a one-cent increase in a particular station's price.<sup>39</sup> As each station's price in our model can be decomposed into a time-varying component  $\nu_{jt}$  and a price reputation component  $\psi_j$ , separate elasticities of demand can be constructed for changes in each price component.

Table 5 summarizes each station's own-price elasticities based on the parameter estimates in Table 4 for the learning model and the no-learning model. In the learning model, the average of a station's own-price elasticity with respect to a change in the time-variant price is -8.36. In contrast, the own-price elasticity with respect to the station's price reputation is -24.73. In other words, consumers are approximately three times more responsive to a change in price reputation than to a change in time-variant price. Two factors contribute to the considerable difference in price elasticities. First, a change in the time-variant price is unknown to consumers prior to search, whereas a change in the price reputation is known ex-ante. Consequently, an increase in the time-variant price at a station can only affect the purchase decisions for consumers who have driven by the station and have not purchased from a previous station. On the other hand, an increase in the station's price reputation may cause more consumers to purchase at earlier stations along their travel route, even before passing that station. Second, when consumers are uncertain about the current price level in the market, a relative price change at a station is confounded by changes in price levels, reducing consumers' responsiveness. They will be less likely to substitute away from a station charging an unexpectedly high price because of the possibility that it reflects an increase in the entire price distribution rather than a relative increase in the station's price. In contrast, consumers will respond more strongly to an increase in a station's price reputation, knowing that it represents a relative deviation from the broader price distribution.

The importance of learning is also highlighted by comparing with the own-price elastic-

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<sup>39</sup>A large proportion of the highway stations' gasoline transactions likely comes from the outside drivers driving past the city via the interstate highway. Because we do not model these passing drivers' purchase decisions, we exclude the highway stations from the own- and cross- elasticity analysis.

ities from the no-learning model in Panel (b). In this model, the average demand elasticity with respect to price reputation is only 19.7% larger than the elasticity with respect to time-variant prices. When consumers do not learn, they do not adjust their prior beliefs about the price level as they observe new prices. Therefore, any observed price change at a station is believed to be specific to that station.

## Spatial Competition and Cross-Price Elasticities

Estimating a structural model of search with learning also provides a useful framework to examine the nature of spatial competition in the market. The stations in our sample exhibit substantial variation in both their characteristics and their locations within the route network. These differences generate considerable variation in own-price elasticity across stations. The estimated station-average own-price elasticities reported in Table 5 Panel (a) range from -20.95 to -2.23, with a standard deviation of 4.27.<sup>40</sup> Stations with very elastic demand tend to face competition from similar stations located nearby. In contrast, stations with the least elastic demand often share little common traffic with other stations or have very different characteristics.

Table 6 provides a more complete picture of the degree of spatial differentiation between each pair of stations in our sample. In addition to the estimated cross-elasticity between station pairs,  $\widehat{\frac{\partial Q_i}{\partial p_j} \frac{p_j}{Q_i}}$ , summary statistics are also reported for the driving distance and the share of common traffic between the stations. Recall from Section 4 that we define Common Traffic as the proportion of station  $i$ 's passing traffic (in all directions) that has previously passed station  $j$  on their travel routes. Drivers driving along travel routes where station  $j$  is downstream to station  $i$  are not included in this Common Traffic calculation because price changes at station  $j$  are unknown to them when visiting station  $j$ .

Not surprisingly, most station pairs have virtually zero cross-price elasticities. After all, more than half of all station pairs have less than 1 percent common traffic share. Only 10 percent of the station pairs, typically involving a station's 4 or 5 closest competitors, have cross-price elasticities larger than 0.15. This makes sense given that 90 percent of station

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<sup>40</sup>Wang (2009) finds similar station level price elasticity. He estimates an own-price elasticity of -18.77 for a station locates right next to its closest competitor and -6.20 for a station whose closest competitor is 4.2 km away.

pairs are over two miles away from each other and have a common traffic share of less than 16 percent. However, some stations do compete intensively. The top 2.5 percent of the pairs have cross-price elasticities above 1.5 and have common traffic shares of greater than 35.8 percent.

Interestingly, allowing for consumer learning can create situations in which competing stations have negative cross-price elasticities. To illustrate, consider three stations, A, B, and C, that share traffic along a particular street as depicted in Figure 5. If consumers traveling east observe a high price at station C, they will not only be less likely to purchase at C but will adjust upward their (posterior) expectations of the general price level. As a result, more of these consumers will purchase at B, whose price now looks more attractive, leaving fewer potential customers for station A. Hence, station A may have a negative cross elasticity of demand with respect to the price at station C. In our learning model, approximately 17.4 percent of station pairs are estimated to have negative cross-price elasticities. In contrast, the more restrictive model with no learning generates all non-negative cross-price elasticity estimates.

Next, we examine how the estimated cross-elasticities between stations vary with geographic and product differentiation. Whereas most studies of gasoline competition rely on simple measures like driving distance to account for geographic differentiation, our traffic flow data allow us to more directly capture connectedness within the travel network using the amount of traffic the stations have in common. The similarity in station characteristics is also likely to influence substitution patterns. In our search model, the ex-ante known mean utility of a station captures its expected attractiveness, reflecting both its characteristics and average price level. If the mean utilities of two stations are sufficiently different, price changes are unlikely to change the stations' utility ranking on a particular day. As a result, consumers driving past these two stations are unlikely to purchase from the less desirable station even when its price is unexpectedly low.

In Table 7, estimated cross-price elasticities for each station pair are regressed on the absolute difference in mean utility of the stations,  $|\hat{V}_i - \hat{V}_j|$ , as well as various measures of the stations' proximity within the travel network. In all specifications, the absolute difference in mean utility has a statistically significant negative coefficient, confirming that

consumers are more likely to substitute between similar stations. The estimates in Column (1) also show that cross-price elasticities generally decline as the driving distance between stations increases, but this relationship becomes insignificant once common traffic measures are included (in Columns 2 through 3). In fact, Column (2) suggests that the traffic share explains a considerable fraction of the variation in substitution patterns between stations. For example, when an additional 10 percent of station  $i$ 's passing traffic has previously driven past station  $j$ , the cross-price elasticity between the two stations increases by 0.43.

Because some left-turns are costly, the ease with which shared traffic can access station  $i$  may impact its cross-price elasticity of demand with respect to an upstream station  $j$ . For this reason, we decompose our Common Traffic measure into two variables based on the traffic's ease of access to station  $i$ . Common Traffic Easy Access measures the share of station  $i$ 's passing traffic that has previously passed station  $j$  and can visit station  $i$  with no additional cost; that is, station  $i$  is on the same side as the traffic or is on the opposite side with an easy left-turn. Correspondingly, Common Traffic Costly Left-Turn measures the share of station  $i$ 's passing traffic that has previously passed station  $j$  and can only visit station  $i$  by a costly left-turn. Indeed, the regression result in Column (3) suggests that cross-price elasticities between stations are significantly higher when the common traffic does not have to make a costly left-turn to access station  $i$ .<sup>41</sup> Given the share of common traffic, station  $i$ 's cross-price elasticity with respect to an upstream station  $j$  is about three times larger when the common traffic can visit station  $i$  easily than with a left-turn cost.

## 8 Biased Priors and Asymmetric Price Pass-through

Lewis (2011), Yang and Ye (2008), and Tappata (2009) each present theoretical models illustrating why cost increases may be passed through more quickly than cost decreases when searching consumers do not know the true price distribution, and Lewis (2011) and Lewis and Marvel (2011) offer reduced-form empirical evidence consistent with such theories. Estimating a structural model of search with learning allows us to more systematically demonstrate the mechanisms through which imperfect knowledge of the price distribution

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<sup>41</sup>We also considered the ease of access for the price-change station  $j$ . However, we did not find statistically differentiated effects.

generates asymmetric pass-through.<sup>42</sup>

Fluctuations in own-price elasticity ultimately drive the fluctuations in price-cost margins associated with asymmetric pass-through (like that illustrated in Figure 1). Indeed, the estimated own-price elasticity on an average day with rising prices is -8.98 compared to the own-price elasticity of -7.75 on an average day with falling prices. This asymmetry in the elasticity implies that the margin is on average 16% smaller when prices are rising than when prices are falling.

The own-price elasticity faced by stations in our model varies over time in response to consumers' prior beliefs about price levels, biased by past prices. Sharper increases or decreases in the price level potentially contribute to a more considerable bias in prior beliefs. To capture this effect, we regress simulated own-price elasticities at the station-day level on the difference between the current price level and the price level seven days ago, controlling for station fixed effects. We refer the coefficient on the change in price level as the *elasticity asymmetry coefficient*. The results based on our full model with learning (Table 8, Column 1) suggest that, when the current price level is 10 cpg higher than in the past, the resulting downward bias in the initial prior increases the own-price elasticities by 0.92 in absolute value. In contrast, an upward bias in the initial prior resulting from higher past prices leads to less elastic demand. These fluctuations in own-price elasticity demonstrate that our learning model with biased priors can rationalize the commonly-observed *rockets and feathers* pattern of asymmetric price pass-through. On the other hand, a similar examination of own-price elasticity estimates from the restricted version of the model with no bias and no learning (shown in Table 8, Column 2) reveals that the restricted model is unable to predict the asymmetric demand response observed in the data.

Within our model, two important mechanisms explain why price changes asymmetrically impact demand elasticities in the presence of biased priors. First, consumers driving along a travel route choose from different stations or postpone a purchase because of heterogeneous postponement costs. How a station's price change affects purchase decisions at the station is captured by the tradeoff between the benefit of postponing and the postponement cost. When prior beliefs are biased downward due to rising prices, a marginal

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<sup>42</sup>In this article, we focus on the demand side responses. As a station is on multiple routes, the complex route structures generate interesting supply-side interactions. We leave this for future work to explore.

increase in a station’s price will cause more consumers to keep searching because they are overly optimistic about the probability that the prices on a future trip are better than the current offer. In contrast, when prior beliefs are biased upward, a marginal increase of a station’s price causes more consumers to purchase because they believe they are unlikely to find a more attractive price. The second important mechanism impacting elasticities is the change in sales volumes that results from prior bias. When prior price beliefs are biased downward, more consumers will choose to postpone purchase, and current sales will be lower than usual. Therefore, the additional consumers a station can attract by cutting price will now represent a larger percentage increase in current sales, implying a higher own-price elasticity of demand.<sup>43</sup>

We further illustrate how fluctuations in own-price elasticity relate to prior bias,  $\pi$ , and the prior uncertainty,  $\alpha_0$ , by using our model to simulate a series of counterfactuals. First, to investigate the importance of biased expectations, we vary the degree to which consumers’ priors of the current price distribution are biased toward past price levels. While holding the other parameters constant at their estimated level, the prior bias parameter is assigned various values ranging from  $\pi = 0$ , where the prior distribution is centered around the actual price level, to  $\pi = 1$ , where the prior is centered around the previous period’s price level. We simulate the predicted own-price elasticities for each prior bias parameter value and then regress these elasticities on the change in the price level from the previous week, mirroring the analysis from Table 8. The elasticity asymmetry coefficient and 95% confidence interval from each regression are plotted as a function of the prior bias parameter in Panel (a) of Figure 6. The figure also plots the estimated average own-price elasticity for each of the prior bias values.

As shown in the figure, the average own-price elasticity stays relatively flat at around -8 as the prior bias parameter varies. In contrast, as prior bias increases, the own-price demand elasticities stations face when prices are rising compared to when prices are falling become more asymmetric, as suggested by the more negative elasticity asymmetry coefficient. As a result, the degree of demand asymmetry as a share of the average own-price elasticity

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<sup>43</sup>Other factors also influence price elasticities, including the density of the postponement cost distribution, route structure, and neighboring stations’ prices. These complicating factors motivated our choice to present a descriptive analysis of demand response asymmetry rather than explicitly demonstrating these relationships within the theoretical model.

increases in the prior bias. More specifically, when consumers have a rational expectation ( $\pi = 0$ ), there is no statistically significant demand asymmetry.<sup>44</sup> On the other hand, when consumers formulate their prior belief entirely on the past prices ( $\pi = 1$ ), the demand asymmetry coefficient is -13.10. On a day when prices are rising and the current price level is 10 cpg higher than last week's price level, the predicted margin would be  $-13.10 * 0.1 / -8 = 16\%$  lower than the margin on an average day in our sample.

A similar counterfactual analysis can be used to evaluate how the degree of certainty consumers attribute to their prior beliefs asymmetrically impacts own-price elasticity following price increases and decreases. When consumers are more certain about their prior beliefs, they place a greater weight on their priors and less weight on newly observed prices when formulating expectations, leading to greater elasticity of demand. For example, consumers are more likely to purchase when encountering a price they think is low because they will be more certain about its low relative position within the price distribution and place little value in the opportunity to continue learning from additional price observations. In addition, because the prior bias is assumed to remain at its estimated value of 0.67, placing additional weight on one's priors allows this bias to generate more persistent differences in consumers' search behaviors when prices are rising and falling, resulting in a more asymmetric response of own-price elasticities.

In Panel (b) of Figure 6 the average simulated own-price elasticity and the asymmetry in that elasticity are plotted for different values of the prior weight parameter. First, the average own-price elasticity grows in absolute value and margins decrease when consumers place a higher weight on their prior beliefs. This is consistent with the theory that consumers become more responsive to price changes when they are more certain about their prior beliefs. Additionally, as prior weight increases, own-price demand elasticities appear to become more asymmetric between periods of increasing and decreasing prices, though the relationship is not monotone. More specifically, as prior weight increases from 0.1 to 81, the demand asymmetry changes from -6.3 to -18.4. However, the demand asymmetry as a share of the average own-price elasticity remains relatively stable. As a result, when the current price level is 10 cpg higher than the last week's price level, the estimated mar-

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<sup>44</sup>The asymmetry is not mathematically zero because the own-price elasticity is simulated based on the observed prices.

gins will be 10 percent lower than the margin on an average day, regardless of the prior uncertainty.

## 9 Conclusion

This article has estimated a dynamic search model with learning where consumers sequentially search for lower gasoline prices in a predetermined order following their travel routes. We allow consumers to be uncertain about the price distribution and hold prior beliefs that may be biased by prices observed during previous purchases. Traffic flow data are used to construct an empirical distribution of search sequences in the market. This novel approach allows us to identify the consumer learning process, postponement costs, and ex-ante seller differentiation using only market share data. We find that consumers place significant weight on past prices when formulating their prior beliefs. However, consumers are relatively uncertain about these prior beliefs. As a result, any initial bias in consumers' expectations diminishes quickly as they update their price beliefs based on new price observations.

By incorporating the consumer learning process, we relax one of the crucial assumptions of standard search models—the assumption that searching consumers are aware of the true price distribution. Prior uncertainty and prior bias are both essential features in the retail gasoline market, as volatile prices make it difficult for consumers to know the true price distribution with any certainty. Consequently, consumers are likely to formulate their expectations of prices based on prices observed in the recent past. More importantly, we systematically demonstrate how prior uncertainty and prior bias can cause demand elasticities to respond asymmetrically to price increases and decreases. This asymmetric demand response offers a direct mechanism to explain why firms pass through positive cost changes more quickly than negative cost changes—a widely observed phenomenon that cannot be explained by search frictions alone. Our results suggest that price fluctuations will have a larger and more asymmetric impact on demand elasticities when consumers rely more heavily on past prices in forming their priors and when consumers place a heavier weight on these priors as they search for gasoline along their travel route.

The use of travel patterns to simulate unobserved search sequences is grounded in the

observation that consumers are likely to search for and purchase gasoline during everyday driving rather than making dedicated trips to purchase gasoline. In addition to the identification of the consumer learning process, our approach has other advantages. First, it allows us to introduce ex-ante vertical differentiation of stations without suffering from the curse of dimensionality. Second, we use the observed traffic flows to replace the random sampling assumption, allowing us to estimate more realistic substitution patterns that depend on the amount of traffic stations share. Although the integration of travel patterns in a search model is most relevant to the retail gasoline market, we envision its applications in other markets. In cases where sellers have physical addresses, such as in a shopping mall, travel patterns naturally constrain the search order. Even for sellers without physical addresses, the order of visits can be affected by constraints such as a webpage layout.

Our article opens up several avenues for future research. We think the most important is the modeling of the supply side decision. The pricing equilibrium arising in the ordered search environment is likely to be quite different from the equilibrium of a random-search model. Arbatskaya (2007) develops a price equilibrium for a row of sellers facing consumers who travel in one direction. However, pricing decisions in the retail gasoline market are more complicated, as stations are located on multiple travel paths with consumers driving in different directions and passing different sets of competitors. Consequently, the demand at a station, as the sum of the residual demand along each search route, is kinked. Spatial differentiation, together with imperfect price information, creates interesting price dynamics, which we leave for future work to explore. Also, asymmetric cost pass-through is often regarded as anti-competitive and harmful to consumers. A supply-side model would enable researchers to answer important welfare questions. For example, a counterfactual analysis could examine how much consumers would benefit from being informed about the actual price distribution and, therefore, facing a market with no asymmetric search intensity and no asymmetric cost pass-through.

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Table 1: Summary Statistics of the Station Characteristics

	Obs.	Mean	SD	25%	50%	75%
<i>Panel (a): All Stations</i>						
Avg. Price (\$)	46	1.61	0.06	1.58	1.59	1.68
Avg. Quantity (gl.)	33	978.09	1207.78	235.76	396.09	1525.02
Major Brands	46	0.37	0.49	0.00	0.00	1.00
Number of Islands	46	3.59	1.73	2.00	3.00	5.00
Easy Left-Turns	46	0.26	0.44	0.00	0.00	0.75
No Left-Turns	46	0.28	0.46	0.00	0.00	1.00
Direct Traffic (1,000s)	46	11.52	4.99	8.32	10.66	15.09
<i>Panel (b): Small Stations</i>						
Avg. Price (\$)	25	1.63	0.06	1.59	1.61	1.69
Avg. Quantity (gl.)	19	379.58	372.86	206.64	253.93	376.10
<i>Panel (c): Large-Format Stations</i>						
Avg. Price (\$)	5	1.57	0.01	1.56	1.57	1.58
Avg. Quantity (gl.)	5	2947.87	1508.80	2217.85	2239.35	2572.08

Table 2: Summary Statistics of Relative Price and Price Level

	Obs.	Mean	SD	Min	Max
<i>Relative Price Changes</i>					
$\nu_{jt}$	23732	0.000	0.032	-0.167	0.169
<i>Price Level Changes</i>					
$\gamma_t$	529	1.622	0.249	1.084	2.042
$abs(\Delta\gamma_t)$	528	0.008	0.012	0.000	0.108
$abs(\Delta\gamma_{t-7})$	522	0.045	0.037	0.000	0.186

Table 3: Descriptive Evidence of Price Level Uncertainty

	(1)	(2)
log(Own Price)	-2.649*** (0.144)	-2.938*** (0.158)
log(1st Neighbor Price)	1.510*** (0.148)	1.259*** (0.158)
log(2nd Neighbor Price)		0.650*** (0.144)
log(Past Price Level)	1.192*** (0.108)	1.125*** (0.108)
$R^2$	0.93	0.93
Observations	15985	15985

**Notes:** The dependent variable is the logarithm of the daily transaction volume at each station. We control for station fixed effects, day of week fixed effects, and month of sample fixed effects in all specifications. Robust standard errors are in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

Table 4: Estimation Results

	(1) Learning		(2) No-Learning	
	Coeff.	SE	Coeff.	SE
<i>Prior</i>				
Bias ( $\pi$ )	0.667	(0.065) <sup>***</sup>	0.015	(0.037)
Learning ( $\alpha_0$ )	0.302	(0.079) <sup>***</sup>		
<i>Station Attributes</i>				
Major Brand	0.516	(0.045) <sup>***</sup>	0.096	(0.080)
Retail Brand 1	-0.139	(0.035) <sup>***</sup>	-0.158	(0.058) <sup>***</sup>
Retail Brand 2	-0.050	(0.027) <sup>*</sup>	-0.182	(0.038) <sup>***</sup>
Small-Sized Station	-1.168	(0.040) <sup>***</sup>	-1.495	(0.089) <sup>***</sup>
Large-Format Station	0.704	(0.038) <sup>***</sup>	1.044	(0.064) <sup>***</sup>
Left-Turn Cost	1.087	(0.056) <sup>***</sup>	1.436	(0.112) <sup>***</sup>
<i>Postponement Cost</i>				
Constant ( $\mu_c$ )	-0.551	(0.045) <sup>***</sup>	-0.257	(0.058) <sup>***</sup>
$R^2$		0.888		0.869

**Notes:** The number of observation is 15985. The day of week and month of sample fixed effect estimates are omitted from the table. The R-squares show the fit of the non-highway stations. Bootstrapped standard errors are in parentheses. <sup>\*\*\*</sup> $p < 0.01$ , <sup>\*\*</sup> $p < 0.05$ , <sup>\*</sup> $p < 0.1$ .

Table 5: Summary Statistics of the Station Average Own-Price Elasticity Estimates

	Obs.	Mean	SD	Min	50%	Max
<i>Panel (a): Learning</i>						
Price ( $\tilde{p}_{jt}$ )	37	-8.36	4.27	-20.95	-7.52	-2.23
Price Reputation ( $\psi_j$ )	37	-24.73	12.54	-58.30	-21.70	-7.59
<i>Panel (b): No Learning</i>						
Price ( $\tilde{p}_{jt}$ )	37	-15.06	6.23	-28.29	-14.05	-3.11
Price Reputation ( $\psi_j$ )	37	-18.03	7.21	-37.50	-16.51	-3.99

Table 6: Summary Statistics on Cross-Price Elasticities and Measures of Spatial Differentiation Between Stations

	Obs.	Mean	SD	2.5%	10%	50%	90%	97.5%
Cross-Elasticity	1665	0.150	1.193	-0.172	-0.002	0.000	0.151	1.539
Driving Distance	1665	5.526	2.979	0.883	2.000	5.173	9.647	12.597
Common Traffic	1665	0.050	0.096	0.000	0.000	0.009	0.160	0.358

Table 7: Regression Results of Estimated Cross-Price Elasticities on Distance Measures Between Stations

	(1)	(2)	(3)
Driving Distance	-0.067*** (0.015)	0.000 (0.004)	-0.001 (0.005)
Abs. Mean Utility Distance	-0.061*** (0.022)	-0.082*** (0.019)	-0.093*** (0.020)
Common Traffic		4.304*** (0.941)	
Common Traffic Easy Access			5.161*** (1.159)
Common Traffic Costly Left-Turn			1.768*** (0.619)
Constant	0.582*** (0.107)	0.013 (0.051)	0.031 (0.053)
$R^2$	0.03	0.12	0.14
Observations	1665	1665	1665

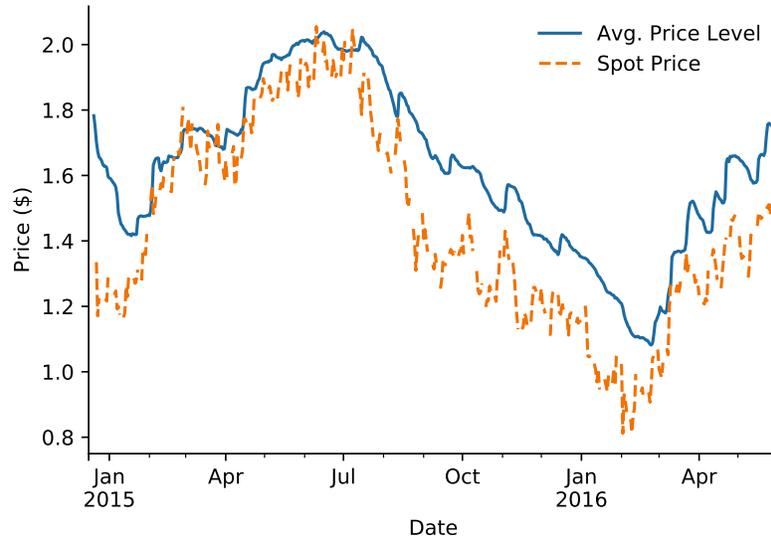
**Notes:** The dependent variable is the cross-price elasticity. Standard errors clustered at the station level are in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

Table 8: Regression Results of Own-Price Elasticities on Past Prices

	(1) Learning	(2) No Learning
$\Delta\gamma_{t-7}$	-9.178** (3.854)	-0.502 (4.469)
$R^2$	0.21	0.18
Observations	18726	18726

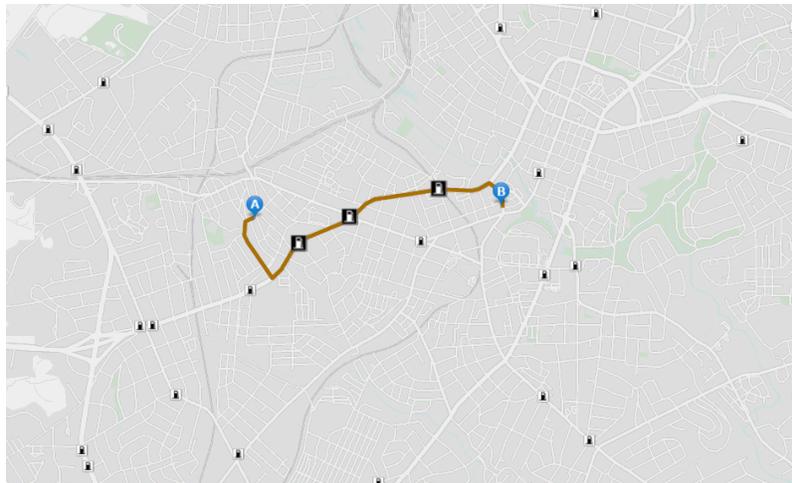
**Notes:** The dependent variable is the own-price elasticity. Standard errors clustered at the station level are in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

Figure 1: Average Retail Gasoline Price Level and Wholesale Cost



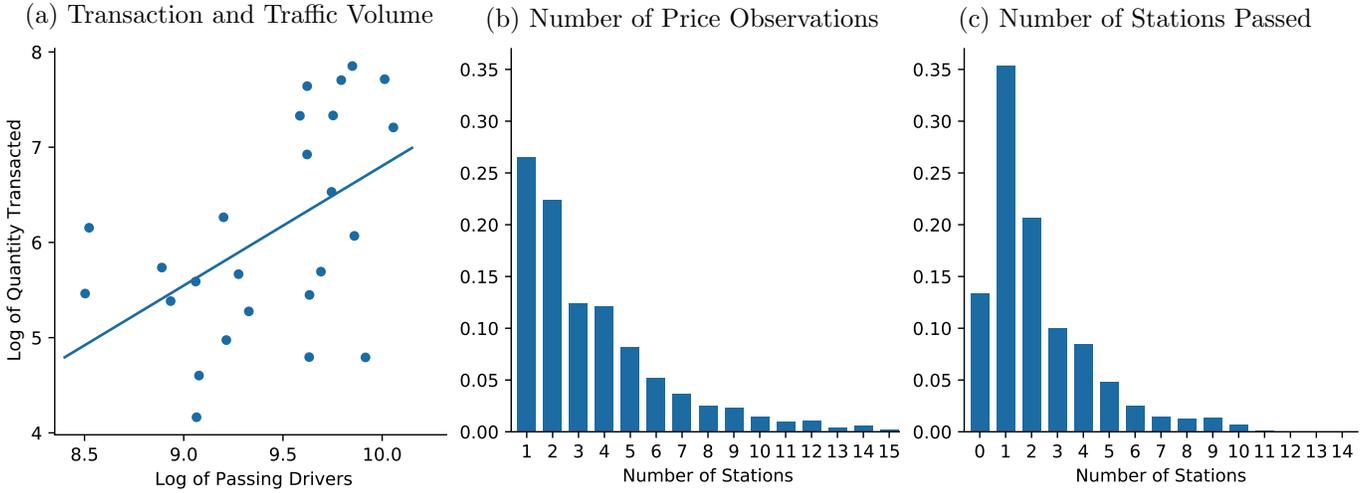
**Note:** This figure plots both the average gasoline price in our sample before federal and state taxes are applied and the Gulf Coast regular spot price as a measure of wholesale cost of retail gasoline.

Figure 2: A Travel Route with Stations Passed



**Note:** This figure represents the road network of a random location. The driving time is less than 5 minutes.

Figure 3: Station Traffic Characteristics



**Note:** The slope of the log-linear fitted line in Panel (a) is 1.26, significant at 5 percent level.

Figure 4: Proportion of Stations Changing Price From Previous Day

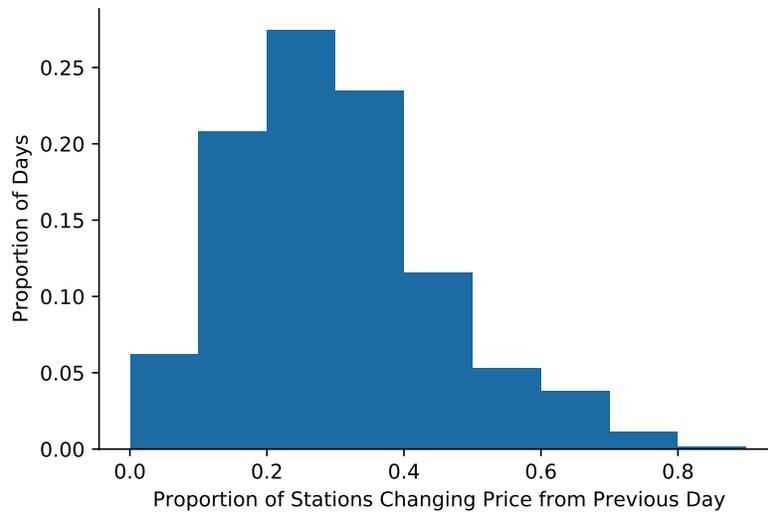


Figure 5: Competing Stations Along a Hypothetical Travel Route

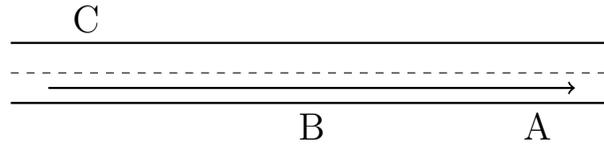
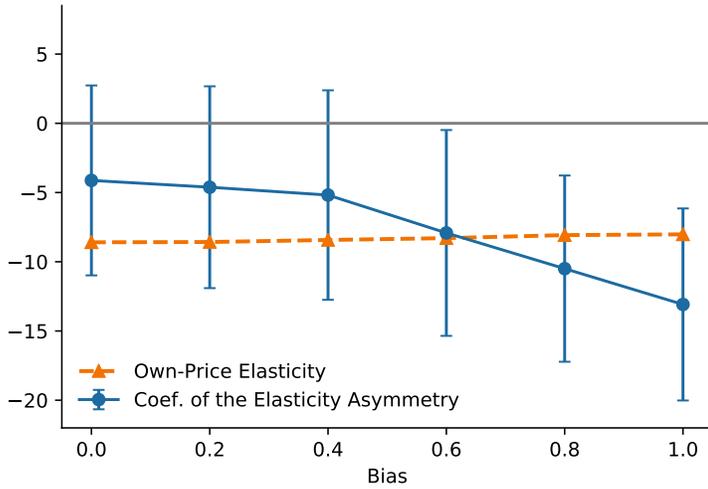
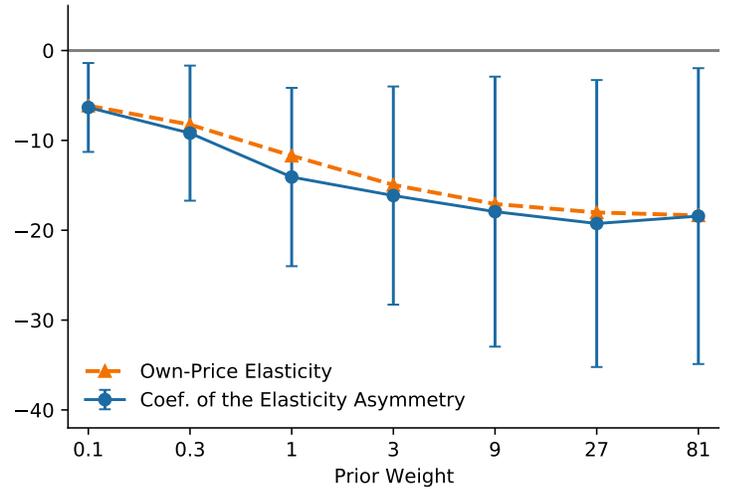


Figure 6: Prior Uncertainty, Prior Bias, and Asymmetry in Own-Price Elasticity

(a) Prior Bias and Elasticity Asymmetry



(b) Prior Uncertainty and Elasticity Asymmetry



# Appendix

## A Derivation of Predictive Distribution

Conditional on a perceived price level  $m$ ,  $x$  is normal with mean  $m$  and a known variance  $\sigma^2$ ,

$$h(x|m) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}.$$

Because there is uncertainty about  $m$ , and the prior belief is expressed as a normal distribution, where  $m \sim N(\mu, \sigma_\mu^2)$ , that is

$$h(m) = \frac{1}{\sigma_\mu\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{m-\mu}{\sigma_\mu}\right)^2}.$$

The predictive distribution is then

$$\begin{aligned} h(x) &= \int h(x|m)h(m)dm \\ &= \frac{1}{\sqrt{2\pi(\sigma^2 + \sigma_\mu^2)}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2 + \sigma_\mu^2}}. \end{aligned}$$

## B Proof of Proposition 1

The transition of the continuation value of search is given by Equation 12 and 11. The Contraction Mapping Theorem applies, and therefore a policy function exists, and it is unique. We guess that  $Z_0(\mu, c_i|\theta) = z_0(c_i) - \mu$ . We backward induct and verify that the continuation value of search is linear additive in  $\mu$  at any station along any route, and consequently verify that the  $Z_0(\mu, c_i|\theta)$  as a weighted average of the continuation value of search at the start of each route minus the postponement cost is linear additive in  $\mu$ .

We suppress  $\theta$  for notation simplicity. We substitute  $Z_0(\mu, c_i) = z_0(c_i) - \mu$  in Equation 11

to obtain

$$\begin{aligned}
Z_{rN_r-1}(\mu, c_i) &= \int \max \left\{ V_{rN_r} - x_{rN_r}, z_0(c_i) - \frac{x_{rN_r}}{\alpha_0 + N_r} - \frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r} \mu \right\} dF_{rN_r-1}(x_{rN_r}) \\
&= \int_{-\infty}^{\frac{\alpha_0 + N_r}{\alpha_0 + N_r - 1}(V_{rN_r} - z_0(c_i)) + \mu} (V_{rN_r} - x_{rN_r}) dF_{rN_r-1}(x_{rN_r}) \\
&\quad + \int_{\frac{\alpha_0 + N_r}{\alpha_0 + N_r - 1}(V_{rN_r} - z_0(c_i)) + \mu}^{+\infty} \left( z_0(c_i) - \frac{x_{rN_r}}{\alpha_0 + N_r} - \frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r} \mu \right) dF_{rN_r}(x_{rN_r}) \\
&= V_{rN_r} \Phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) - \mu \Phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) + \sigma \sqrt{\frac{\alpha_0 + N_r}{\alpha_0 + N_r - 1}} \phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) \\
&\quad + z_0(c_i) \left( 1 - \Phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) \right) - \frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r} \mu \left( 1 - \Phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) \right) \\
&\quad - \mu \frac{1}{\alpha_0 + N_r} \left( 1 - \Phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) \right) - \sigma \frac{1}{\alpha_0 + N_r} \sqrt{\frac{\alpha_0 + N_r}{\alpha_0 + N_r - 1}} \phi \left( \frac{V_{rN_r} - z_0(c_i)}{\sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}}} \right) \\
&= -\mu + z_0(c_i) + \sigma \sqrt{\frac{\alpha_0 + N_r - 1}{\alpha_0 + N_r}} (\zeta_{rN_r} \Phi(\zeta_{rN_r}) + \phi(\zeta_{rN_r})) \\
&= -\mu + z_{rN_r-1}(c_i),
\end{aligned}$$

where  $\zeta_{rn+1} = \frac{V_{rn+1} - z_{rn+1}(c_i)}{\sigma \sqrt{\frac{\alpha_0 + n}{\alpha_0 + n + 1}}}$ . Follow the same recursive relationship, we obtain Equation 13,

$$z_{rn}(c_i) = z_{rn+1}(c_i) + \sigma \sqrt{\frac{\alpha_0 + n}{\alpha_0 + n + 1}} \cdot (\zeta_{rn+1} \cdot \Phi(\zeta_{rn+1}) + \phi(\zeta_{rn+1})).$$

At the first station  $Z_{r1}(\mu, c_i) = -\mu + z_{r1}(c_i)$ ,

$$\begin{aligned}
Z_0(\mu, c_i) &= -c_i + \sum_{r' \in \mathbb{R}} \lambda_{r'} \int \max \left\{ V_{r'1} - x_{r'1}, z_{r1}(c_i) - \frac{x_{r'1}}{\alpha_0 + 1} - \frac{\alpha_0}{\alpha_0 + 1} \mu \right\} dF_{r0}(x_{r'1}) \\
&= -c_i + \sum_{r' \in \mathbb{R}} \lambda_{r'} \left[ \int_{-\infty}^{\frac{\alpha_0 + 1}{\alpha_0}(V_{r'1} - z_{r'1}(c_i)) + \mu} (V_{r'1} - x_{r'1}) dF_{r0}(x_{r'1}) \right. \\
&\quad \left. + \int_{\frac{\alpha_0 + 1}{\alpha_0}(V_{r'1} - z_{r'1}(c_i)) + \mu}^{+\infty} \left( z_{r'1}(c_i) - \frac{x_{r'1}}{\alpha_0 + 1} - \frac{\alpha_0}{\alpha_0 + 1} \mu \right) dF_{r0}(x_{r'1}) \right] \\
&= -\mu - c_i + \sum_{r \in \mathbb{R}} \lambda_r \left[ z_{r'1}(c_i) + \sigma \sqrt{\frac{\alpha_0}{\alpha_0 + 1}} \cdot (\zeta_{r'1} \cdot \Phi(\zeta_{r'1}) + \phi(\zeta_{r'1})) \right].
\end{aligned}$$

This verifies that  $Z_0(\mu, c_i)$  is indeed linearly additive in  $\mu$ .

## C Monte Carlo Simulation

Figure C.1: Sample Market

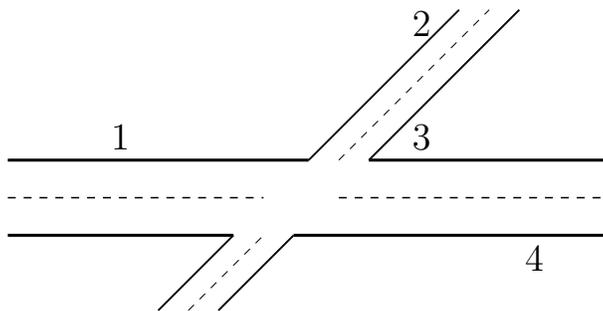


Table C.1: Sample Search Routes

Order			Left-Side			$\lambda$
1	3	4	1	1	0	0.12
4	3	1	1	0	0	0.12
1	3	2	1	0	0	0.10
2	3	1	0	1	0	0.10
2	3	4	0	1	0	0.06
4	3	2	1	0	0	0.06
1	3		1	1		0.05
3	1		0	0		0.05
2	3		0	1		0.04
3	2		0	0		0.04
1			0			0.05
1			1			0.05
2			0			0.10
3			0			0.01
3			1			0.01
4			0			0.02
4			1			0.02

We use Monte Carlo simulations to verify that our estimation approach can recover the true parameters of the learning process, postponement cost, and product differentiation. The experiment procedure is as follows. We consider a sample market that consists of a small set of four stations and 17 travel routes. Figure C.1 provides a layout of this market, and Table C.1 shows the route structures. For example, the first row of Table C.1 suggests

Table C.2: Estimation Results from Monte Carlo Experiments

Variable Variable	True Values	Estimated Coeff. (SE)
Bias	0.6	0.567 (0.067)
Prior Weight	0.5	0.524 (0.099)
Brand	0.5	0.501 (0.015)
Left-Turn Cost	1.0	1.014 (0.135)
Postponement Cost	-0.5	-0.519 (0.051)

that 12 percent of the total traffic drives past three stations in the order of Station 1, 3, and 4, with Station 1 and 4 on the opposite side of the street. Note that we assume Station 4 is located at a street section where left-turns can be easily made, and thus it is at the same street side as the travel direction for all of its passing traffic. Station 1 and 2 are branded stations, and Station 3 and 4 are generic stations (base group). Recall that we decompose the price for 10 gallons of gasoline into three parts (Equation 1). In the exercise, the stations' average price deviations from the market average are 0.3, 0.5, -0.3, and -0.5, respectively, price residuals are randomly drawn from a normal distribution with a mean zero and a variance of 0.1, and the daily average price level follows a random walk with a starting value of 20 and a normal stochastic component with a mean zero and a standard deviation of 0.15. Prices are simulated for 500 days. We compute the market shares based on Equation 20, and we further add normal prediction errors with a mean zero and a standard deviation of 0.03 to the simulated market shares.<sup>45</sup> All of the parameters applied to simulate the data are chosen to mimic the characteristics of our data sample.

Table C.2 presents the results. Column (1) provides the true parameter values used to generate the data, and Column (2) shows the mean and the corresponding standard deviations of the recovered parameters from 100 replications. We can see that all of the recovered parameters are close to and within one standard deviation of the true values. The numerical experiments confirm that our estimation strategy can separately identify the unknown parameters of the learning model.

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<sup>45</sup>The average  $R^2$  over all replications is 0.908.